

Abstract Algebra
Math 521A
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Review for second exam

Rings

- Know the definitions:
 - ring, commutative, identity, field;
 - unit, zero divisor, characteristic;
 - homomorphism, isomorphism.
- Know how to:
 - Prove that a subset of a ring is a subring, (or show that it isn't).
 - Prove that a function is a homomorphism, or isomorphism (or show that it isn't).
 - Show that two rings can't be isomorphic, because they have some difference in structure.
 - Identify the units and zero divisors in a ring.
- Know how to construct new rings from old and to compute in these rings.
 - The Cartesian product of rings R and S is a ring $R \times S$.
 - The 2×2 matrices over a ring R form a ring, which we write $M(R)$.
 - We also have the polynomial ring, $R[x]$ over a ring R .

Polynomial rings over a field F

- Know the special properties of $F[x]$, and that it is similar to \mathbb{Z} .
 - Division theorem.
 - Euclidean algorithm.
 - Prime iff irreducible.
 - Unique factorization.
- Know the relationship between roots and factors.
 - In $F[x]$, the remainder when $f(x)$ is divided by $(x - a)$ is $f(a)$.
So $x - a$ is a factor of $f(x)$ iff a is a root of $f(x)$.
 - A polynomial of degree d has at most d roots.
 - Know how to test whether a polynomial in $\mathbb{Z}_p[x]$ of degree 2 or 3 is irreducible.