

# Abstract Algebra B

## Math 521B

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Review for third exam

- Be able to define precisely the following terms. Be careful about the logic in the definition!
  - transcendental element, algebraic element, minimal polynomial (Thm. 10.6).
  - algebraic extension, simple extension, finitely generated extension, splitting field.
- Know some standard examples over  $\mathbb{Q}$  and  $\mathbb{R}$ .
  - Find the minimal polynomial of some simple examples (e.g.  $\sqrt{2+i}$ ).  
See 10.2 # 1-7, 11,17.
  - Find bases for extensions: (e.g.  $\mathbb{Q}(\sqrt{2}, \sqrt{3})$  over  $\mathbb{Q}$ ).
  - Know the theorems and exercises about the dimension and bases of a composite extension.  
(Thm. 10.4, Thm 10.10, Ex. 10.3 #1-7, 8, 9, 11, 13).
- Know how to work with finite fields.
  - Know the Freshman's rule (see also Ex. 10.6# 10, 12).
  - Know the fundamental theorem for finite fields (as stated in class, but also spread out in Sec. 10.6)
    - \* There exists a field of order  $p^n$  for each prime  $p$  and  $n \in \mathbb{N}$ .
    - \* The field is unique up to isomorphism, since it is the splitting field of  $x^{p^n} - x$ .
    - \* The multiplicative group of a finite field is cyclic.
    - \* Any finite field is a simple extension of a prime field.
  - Given an irreducible polynomial whose root  $\alpha$  generates the multiplicative group of  $\mathbb{F}_{p^n}$ , construct the dictionary between powers of  $\alpha$  and polynomials in  $\alpha$ .
  - Use the dictionary to compute.