

NAME:

Abstract Algebra B

Math 521B

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Review for first exam

- Be able to precisely define the following terms. Be careful about the logic in the definition!
 - Group, subgroup, cyclic group, generators of a group.
 - Order of an element, order of a group.
 - Homomorphism, isomorphisms, automorphism, inner automorphism.
 - Center of a group, centralizer of an element.
 - Normal subgroup.
- Here are the key theorems; be able to use them.
 - Theorem 7.8: the order of an element.
 - Theorem 7.10: properties determining a subgroup.
 - Theorems 7.18, 7.28: cyclic groups.
 - Theorems 7.19 (and related problems, 7.4#9, 12, 23): Properties of homomorphisms.
 - Theorem 7.23, 7.24: Properties of cosets (see my in class version).
 - Theorems 7.26, 7.27: (Lagrange) the order of subgroups of a finite group.
 - Theorems 7.34, 7.36: Properties of normal subgroups and quotients.
 - 7.41, 7.42, 7.43, 7.44: Homomorphism and isomorphism theorems.
 - Let $\alpha : G \rightarrow H$ be a homomorphism. You should be able to prove these.
 - * If B is a subgroup of H then $\alpha^{-1}(B)$ is a subgroup of G . In addition, if B is normal then $\alpha^{-1}(B)$ is normal.
 - * If A is a subgroup of G then $\alpha(A)$ is a subgroup of H . (Caution: If A is normal, $\alpha(A)$ may not be!)
 - You should be able to prove some of the simpler results about abelian groups and the order of an element (Sec. 7.2).
- Know how to work with the standard examples.
 - $(\mathbb{Z}_n, +)$, $(U_n, *)$.
 - $\text{Gl}(2, \mathbb{Q})$, $\text{Sl}(2, \mathbb{Q})$ and the matrix groups over \mathbb{Z}_p for p prime.
 - The symmetric group S_n , the dihedral group D_n .
 - Subgroups of the above, such as the quaternions.
 - Review the classification of groups of order < 8 that we did in class.