

Abstract Algebra B

Math 521B

Michael E. O'Sullivan

Review for second exam

- Normal subgroups and quotient groups
 - Know the first and third isomorphism theorems.
 - Know how to compute in quotient groups.
- Direct products of groups
 - Know how to compute in direct products (componentwise!)
 - Subgroups and direct products.
 - * If M is a subgroup of G and N is a subgroup of H then $M \times N$ is a subgroup of $G \times H$.
 - * When M and N are normal, $M \times N$ is normal.
 - * In this case, $G \times H / M \times N \cong (G/M) \times (H/N)$.
 - Not all subgroups of $G \times H$ are of the form $M \times N$. Give an example.
- Classification of finite abelian groups.
 - Be able to state and use the two main theorems:
The Fundamental Theorem of Finite Abelian Groups,
The Invariant Factor Theorem.
 - Be able to compute the elementary divisors and invariant factors of some given group:
e.g. $\mathbb{Z}_{60} \times \mathbb{Z}_{75} \times \mathbb{Z}_{100}$, or U_{360} .
 - Suppose I give you n . How many abelian groups are there with n elements? How many have 2 invariant factors? What is the largest number of invariant factors possible for this n ?
- The symmetric group, S_n .
 - Transform between cycle notation and function notation. Know the unique factorization theorem (as a product of disjoint cycles).
 - I defined the signature of $\sigma \in S_n$ to be the list of cycle lengths in the unique factorization (in decreasing order). What are the possible signatures for an element of S_n ?
 - Be able to compute with each and compute the order of an element.
 - Be able to state and use the conjugacy theorem.
 - I defined t
 - Generators of S_n : Cor. 7.48, exercises 7.9 #33, 34.
 - Know S_n and A_n for $n \leq 4$ very well.
- Some infinite abelian groups
 - The groups \mathbb{Q}/\mathbb{Z} , \mathbb{R}/\mathbb{Z} .
 - The groups \mathbb{Q}^* , \mathbb{Q}^{**} , \mathbb{R}^* , \mathbb{R}^{**} , \mathbb{C}^* .