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Math 720: Commutative Algebra and Algebraic Geometry

Homework 5 and 6

Here is a list of problems to read/compute/understand.

IV.1	#1-3, 7, 8, 10	Ideals giving empty varieties over a non algebraically closed field.
IV.2	#1-8, 10, 14-17	Properties of radical ideals. Examples of radical ideals. GCD of two polynomials. Radical ideals and Groebner bases.
IV.3	#1-7, 9, 11-15	GCD and LCM of polynomials, and corresponding properties of ideals. Properties of sum, product, intersection of ideals, and radicals of these. Comaximal ideals and the CRT. Linear transformations giving homomorphisms of polynomial rings.
IV.4	#1-10, 16	Ideal quotients, saturation.

Turn in these problems on **Th 3/16** :

IV.1	#8, 10
IV.2	#15 (using quotient ring), 17 (read 16 too)
IV.3	#9, 12

Turn in problem A below and these problems on **Th 3/23** :

IV.3	# 11
IV.4	#3, 4, 8
IV.5	#3, 4, 12

Problem A: Let I and J be principal ideals in $k[\overline{x}]$, generated by $p(\overline{x})$ and $q(\overline{x})$. Unique factorization in $k[\overline{x}]$ gives a formula for a generator for $I \cap J$ from the factorizations of p and q (see IV.3#2-5). The formula does not indicate how to actually compute a generator for $I \cap J$, for that we need IV.3 Theorem 11 and elimination. Show in detail how this works: $\Big(t\langle p\rangle + (1-t)\langle q\rangle\Big) \cap k[\overline{x}]$ is generated by LCM (p, q).