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Spring 2017

Math 720: Commutative Algebra and Algebraic Geometry
Test 2 prep

1 Here are three fundamental results about ring homomorphisms and ideals: be prepared to prove them. Let $\varphi : R \longrightarrow S$ be a homomorphism of rings and J an ideal in S.

(a) $\varphi^{-1}(J)$ is an ideal in R.

(b) If J is prime then so is $\varphi^{-1}(J)$.

(c) For R and S polynomial rings over an algebraically closed field k, if J is maximal in S then $\varphi^{-1}(J)$ is maximal in R.

2 Be able to explain high-level steps in an algorithm to compute the following.

(a) Generators for the ideal $I(F(k^n))$ where $F : k^m \longrightarrow k^n$ is a polynomial map.

(b) Generators for the ideal $I \cap J$ for $I, J \subseteq k[x_1, \ldots, x_n]$.

(c) Generators for the ideal $I : J$ for $I, J \subseteq k[x_1, \ldots, x_n]$.

3 Be able to discuss problems on parametrization in Section 3.3 # 3-7.

4 Understand the algebra-geometry correspondence for an algebraically closed field that arises from the Nullstellensatz.

5 Be able to prove basic properties of the sum, product and intersection of ideals in Sections 4.3, 4.4.

6 Be able to use the theorems from Section 4.6 (just over $\mathbb{C}$). Every variety has a unique decomposition into a union of irreducible varieties. There is a one-to-one correspondence between irreducible varieties and prime ideals. Every radical ideal may be written in a unique way as the intersection of prime ideals. Read the proof of Theorem 7.