Mike O'Sullivan
Department of Mathematics
San Diego State University

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Math 627A: Modern Algebra I

Homework I

Problem 1: (see Rotman ex. 8) Let F be a field and denote by D the formal derivative on $F[x]$. Show that D satisfies the sum-rule, the product rule and the chain rule.
[Hint for the chain rule: From the product rule, use induction to derive a rule for the derivative of $g(x)^m$.]

Note We will also write f' for Df

Problem 2: (Rotman ex. 44) Let $f(x) = \prod_{i=1}^{d}(x - a_i) \in F[x]$, where F is a field and $a_i \in F$ for all i. Show that $f(x)$ has no repeated roots if and only if $\gcd(f(x), f'(x)) = 1$.

Problem 3: (Rotman ex. 59) Suppose F is a field of characteristic 0. If $f(x)$ is an irreducible polynomial in $F[x]$ then $f(x)$ has no repeated roots in any extension field of F.

Problem 4: Let $f(x) = \prod_{i=1}^{d}(x - a_i) \in F[x]$, and suppose

$$g(x) = \sum_{i=1}^{d} e_i \prod_{j=1 j\neq i}^{d} (x - a_i)$$

Show that $g(a_i)/f(a_i) = e_i$.

Note: This result applies to an error correction algorithm used in coding theory.

Due: Monday, October 3rd, class time. You are encouraged to work together to analyze and solve the problem, but please write your solutions individually.