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Math 627B: Modern Algebra II
Homework V

Problems 1 and 2. Due Th. 10/25 2012.

Problem 1: Shorties.

(a) Show that the intersection of two normal subgroups of G is normal in G.

(b) If every element in G has order 2 show that G is abelian.

(c) If G has even order then G has an element of order 2.
(Consider the pairing of g with g^{-1}).

Problem 2: A group is *metabelian* when it has a normal subgroup N such that N and G/N are both abelian. A group is *metacyclic* when it has a normal subgroup N such that N and G/N are both cyclic.

(a) Show that S_3 is metacyclic.

(b) Show that A_4 is metabelian but not metacyclic.

(c) Prove that any subgroup of a metabelian group is also metabelian.

(d) Prove that any quotient group of a metabelian group is metabelian. [Look carefully at the proof of the 2nd isomorphism theorem and adapt it to this question.]