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Math 627B: Modern Algebra II
Final Exam Preparation

For the final exam I will choose THREE of the topics listed below and you will be asked to write an essay on TWO of them. You have a certain degree of freedom in your essay, but I may ask a few specific questions on material that is central to the topic.

Please consider the following general guidelines:

- Break your essay into two parts.

  - Part 1: Imagine you are writing for a fellow master's student. Your goal is to explain to them the essential aspects of the topic. State the main theorems, illustrate with examples. You may simply state some results, sketch proofs or give detailed proofs of other results. Your choice of presentation is based on what you think is important, interesting, and possible given your understanding and the time limitation.

  - Part 2: Explore a challenging example or advanced topic. This might be a theorem that we did not cover in the course, a complex example, something you discovered in reviewing during the week, something you read in another source, etc.

- You may make reference to theorems using a shorthand, descriptive name, and you can create your own descriptor for the purposes of the essay (*e.g.* Unique Factorization Theorem, Root-Factor Theorem).

- Enrich your presentation with examples!

- You will not be able to use your notes or a book.

Essay Topics

Topic A: The Fundamental Theorem of Galois Theory concerning Galois field extensions and the corresponding relationship between intermediate fields and subgroups of the Galois group.

Topic B: Finite group theory. The semi-direct product, the Sylow theorems, describing groups by generators and relations (dihedral groups, dicyclic groups, groups of small order).

Topic C: The Galois theory of cyclotomic and radical extensions of $\mathbb{Q}$.

Topic D: Fields in finite characteristic. Focus on finite fields, but if you want to discuss the field $\mathbb{F}_p(t)$, you may. Explain the Galois correspondence for a finite field.