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Math 627B: Modern Algebra I
HW 5

Due: Tuesday 4/19, 2016.

1 Problems

Problem 1: Sixth roots of unity.

(a) Factor $x^6 - 1$ completely over $\mathbb{Q}$.

(b) Show that the splitting field of $x^6 - 1$ is of degree 2 over $\mathbb{Q}$.

Problem 2: Minimal polynomials for elements of $\mathbb{Q}(\sqrt[3]{2})$ (see [A] 3.1#4,5).

(a) Find the minimum polynomial of $\sqrt[3]{2} + 1$ over $\mathbb{Q}$. [There is a clever way that involves little computation, but don't feel obliged to use it!]

(b) Find the minimum polynomial of $\sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$.

Problem 3: Show that $\mathbb{Q}(i\sqrt{5})/\mathbb{Q}$ and $\mathbb{Q}((1+i)\sqrt[4]{5})/\mathbb{Q}(i\sqrt{5})$ are normal, but $\mathbb{Q}((1+i)\sqrt[4]{5})/\mathbb{Q}$ is not normal.

Problem 4: Prove: If H is an intermediate field of the finite Galois extension G/F then H/F is normal if and only if $\sigma(H) \subseteq H$ for all $\sigma \in \mathrm{Aut}(G/F)$.

Problem 5: Let $E = \mathbb{Q}(i, \sqrt{2})$.

(a) Find the minimum polynomial of $i + \sqrt{2}$.

(b) Show this polynomial splits in E.

(c) Show that E is a simple extension of $\mathbb{Q}$.