DISCRETE MATHEMATICS
Math 245
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Practice for the Second Exam

I. Some proofs. Be organized and clear.

a)Let A, B, and C be subsets of a set U. Prove that $(A \cap B) - C = (A - C) \cap (B - C)$. (You may use elements or give an algebraic proof.)

b) If A, B, and C are sets such that $A \subseteq B$, and $A \subseteq C$, then $A \subseteq B \cap C$.

c) If R and S are symmetric relations, $R \cap S$ is a symmetric relation.

d) If R and S are symmetric relations, $R \cup S$ is not necessarily a symmetric relation. (Just give an example.)

II. Classic proofs:

a) Prove that $\sqrt{5}$ is irrational.

b) Prove there are an infinite number of primes.

c) Prove that the sum of a rational and an irrational number is irrational.

III. (50 pt.) Let $A = \{a, b, c, d, e, f\}$. Make up your own relation R on A with 5 elements. Answer the following. For a), b), c), you may use a list, a table, or a directed graph to portray the relation.

a) Find the smallest relation containing R which is reflexive.

b) Find the smallest relation containing R which is symmetric.

c) Find the smallest relation containing R which is transitive.

d) Let S be the smallest equivalence relation containing R. Give the partition induced by S.

e) Let T be the smallest partial order containing R. Draw the Hasse diagram for T.

IV. (30 pt.) For $n = 2, 3, \ldots, 36$ do the following,

a) Draw the Hasse diagrams for D_n.

b) Identify the minimal elements of $D_n \setminus \{1\}$.

V. (30pts.) Consider the equivalence relation S on the set, $\mathbb{Z} \times \mathbb{N}$.

$$(a, b)S(r, s) \text{ provided } as = br$$

a) Prove that S is an equivalence relation.

b) What is the equivalence class of $(3, 1)$? What is in the equivalence class of $(1, 3)$? What is the equivalence class of a general (a, b)?

b) Identify a set that has exactly one representative from each equivalence class.

VI. The *modulo* n relation on Z:

a) Under the the modulo 5 relation, when is $a \in \mathbb{Z}$ related to b?

b) Prove that the modulo 5 relation is an equivalence relation

c) What are the equivalence classes modulo 5?

VII. Fuctions:

a) Give n example of a function from $A = \{a, b, c, d\}$ to $B = \{x, y, z\}$ which is surjective (onto), but not injective (one-to-one).

b) Give an example of a function on A which is neither surjective nor injective.

c) Give an example of a function on $\mathbb{Z}$ which is injective but not surjective (and vice-versa).