

# DISCRETE MATHEMATICS

## Math 245

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Assignment for Ch 10

Due Wed. 11/24/10

1. Epp 10.2 #11 (both editions).
2. Epp 10.2 #44 (both editions).
3. Let  $A = \mathcal{P}(\{a, b, c\})$ , the set of subsets of  $\{a, b, c\}$ . Define a relation on  $A$  by  $XY$  when  $|X| = |Y|$  (here  $X$  and  $Y$  are subsets of  $\{a, b, c\}$  and  $|X|$  is the number of elements in  $X$ .)
  - (a) Prove that  $R$  is an equivalence relation (this is straightforward.)
  - (b) Identify the equivalence classes of  $R$ .
4. Consider the integers,  $\mathbb{Z}$ , with the mod 7 equivalence relation. Give a system of representatives.
5. Consider the "punctured plane," that is  $\mathbb{R} \times \mathbb{R} - \{(0, 0)\}$ . Define an equivalence relation  $R$  by  $(x_1, y_1)R(x_2, y_2)$  when there is some positive real number  $a$  such that  $ax_1 = ax_2$  and  $ay_1 = y_2$ . In other words:  $(x, y)R(ax, ay)$  for all positive reals  $a$ .
  - (a) Find all points  $(x, y)$  such that  $(0, 1)R(x, y)$ . Find all points  $(x, y)$  such that  $(1, 1)R(x, y)$ .
  - (b) Show that  $R$  is an equivalence relation.
  - (c) Briefly state what the equivalence classes are.
  - (d) Find a system of representatives: a set with exactly one element from each equivalence class.
6. Construct the Hasse diagram of  $D_n$  for  $n = 42$ , and  $n = 100$ .
7. For the values of  $n$  in the previous problem, find the maximal and minimal elements of  $D_n - \{1, n\}$ .
8. Construct the Hasse diagram for the divides relation on the set  $\{2, \dots, 10\}$ . Identify the minimal elements and the maximal elements.