# DISCRETE MATHEMATICS
## Math 245

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Expectations for the Final Exam

These lists contain all concepts you should understand and skills you should have aquired to demonstrate your understanding. The lists are by topic, although there is a good deal of interaction among the topics. * indicates Level 2.

I. Understand fundamental logic:

- Know the truth tables for $\wedge$, $\vee$, $\Longrightarrow$, xor and be able to compute truth tables for compound statements.
- Be able to establish logical equivalence using a truth table.
- Know some important equivalences and apply them to every day language and mathematical statements. (See Thm. 1.1.1 and $p \Longrightarrow q \equiv \sim p \vee q$ and $p \Longrightarrow (q \vee r) \equiv (p \wedge \sim q) \Longrightarrow r$).
- (Important!) Compute the negation of a statement, using English sentences, mathematical sentences and formal logic.
- Use existential ($\exists$) and universal ($\forall$) quantifiers correctly. Compute the negation of a quantified expression.

II. Know the basics of set theory!

- Define subset and verify that one set is a subset of another.
- Compute intersection, union, difference, and complement of sets. Define these terms.
- Define and compute the power set of a set, and the Cartesian product of two or more sets. Define and explain what a partition of a set is.
- *Be able to prove one set is a subset of another.

III. Know the basics of number theory.

- Use the definition of *divides* to establish divisibility properties.
- State and use the Quotient-Remainder Theorem (aka Division Theorem).
  - Use division and the Euclidean algorithm to find the gcd of two numbers.
  - Use division theorem to find base $b$ representation.
- Compute base $b$.
- State and use the Unique Factorization Theorem.
- * Know the classic proofs by contradiction.
  - $\sqrt{p}$ is irrational for $p$ prime.
  - The sum of a rational number and an irrational number is irrational.

IV. Functions and relations.

- Know the definitions: Relation, inverse of a relation, function.
  For functions: injective (one-to-one), surjective (onto) and bijective.
- Determine whether a relation is a function, using the definition of function. Similarly, determine if a function is injective, surjective, or bijective.
- Work with lists, tables, arrow diagrams and formulas to define relations.
- Find the inverse of a bijective function (e.g. $f(x) = 2x + 7$).
- Give examples of functions satisfying various properties
  (7.3 #4-10 (2nd Ed.) 7.2 #5-14 (3rd and 4th Ed.)).
- * Be able to prove statements about functions (§7.3 #16-19 4th Ed., §7.4 #16-19 3rd Ed., §7.5 #15-18 2nd Ed.)

V. Relations on a set.

- Know how to use a table, a list of elements or an arrow diagram (also called a directed graph) to represent a relation on $X$. (The arrow diagram for a relation on $X$ is different from that for a relation from $X$ to $Y$).
- Verify or prove that a relation $R$ is reflexive (ditto for symmetric, transitive, antisymmetric, an equivalence relation, or a partial order).
- For a relation $R$ on $A$, be able to find the smallest relation containing $R$ which is symmetric (the symmetric closure). Similarly, find the reflexive closure, the transitive closure, or the smallest equivalence relation containing $R$.
- Know the standard examples of equivalence relations
  (mod $n$, and * 10.3 #18, 19, 22, 23 3rd Ed., 10.3 #15, 16, 19, 20 2nd Ed.).
- Know the standard examples of partially ordered sets: $\leq$ for the integers (or rationals) divides on the integers; $\mathcal{P}(\mathcal{A})$ for a set $A$; $D_n$; (10.5 #16, 17, 18, 19, 20, 21 both Eds.).
- Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

VI. Know the basics of recursion and induction!

- Be able to use summation and product notation.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.
- Verify that a sequence satisfies a recurrence formula.
- Know the formulas for the following sums:
  - The sum of a geometric sequence.
  - The sum of the first $n$ integers.
- * Be able to state the well-ordering principle.
- * Be able to state the principle of induction.

VII. Know how to prove by standard induction! (I won't test strong induction.)

- Given the outline of an induction proof, be able to fill in the details.
- *Write you own induction proof.
  - Use full sentences.
  - State the predicate.
  - Prove the base step.
  - State the assumption for the inductive step.
  - Do the inductive step.

VIII. Know how to count!

- Know the inclusion-exclusion formula and be able to apply it and use a Venn diagram to illustrate.
- Know the 4 ways to choose and the formulas for 3 of them (I won't test "order unimportant, repetition allowed").
- Poker hands (I will describe the hand, and I may give you a strange deck or strange hand).