DISCRETE MATHEMATICS
Math 245

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Suggestions for Preparing for the Final Exam

Logic and Sets

I. Know the fundamentals of logic.

- Use a truth table: To show two statements equivalent, to prove a statement is a tautology or a contradiction, to show that an argument is valid.
- Use disjunctive normal form to create a logic statement having a given truth table.
- Use the logical equivalences we've established to simplify a statement. The main equivalences are Thm. 1.1.1 and $(p \Longrightarrow q) \equiv (\sim p \vee q)$ and $\Big(p \Longrightarrow (q \vee r)\Big) \equiv \Big((p \wedge \sim q) \Longrightarrow r\Big)$.
- Pay particular attention to quantifiers: existential ($\exists$) and universal ($\forall$).
- Know how to negate a statement (Important!).

II. Know how to translate from English to formal logic and vice-versa.

- Standard "or" versus "exclusive or."
- Variety of ways to express a conditional.
- Variety of ways to express universal and existential statements.

III. Sets: Know the definitions!

- Subset. Intersection, union, set difference, complement.
- Power set, Cartesian product, partition.

IV. Be able to:

- Draw a Venn diagram illustrating set operations.
- Use an element argument in a direct proof that one set is a subset of another.
- Use established properties (distributivity, De Morgan's etc.) to algebraically prove new ones.

The Integers, Sequences and Induction

I. Things you should know about the integers and rational numbers:

- Be able to use (and recognize that you are using) commutativity, associativity, the additive and multiplicative identity, the additive inverse (and, for the rationals, the multiplicative inverse), distributivity.

- Be able to use (and recognize that you are using) properties of $<$. For example $a < b$ implies $a + c < b + c$.
- Be able to define prime, composite, divides, floor, ceiling.

II. Know the statements of the following theorems and know how to apply them (as in webworks problems):

- Quotient-remainder theorem.
- The unique factorization theorem.

III. Be able to do these computations.

- Use the Euclidean algorithm to find the greatest common divisor of two numbers.
- Convert an integer (base 10) into another base, and convert from any base into base 10.
- Add in any given base. Construct a multiplication table in a given base. Use a multiplication table to find a product of two numbers in any given base.

IV. Know these standard proofs and proof methods.

- Divisibility results like:
  - Transitivity of divides.
  - If a divides b and a divides c then a divides $b + c$.
  - When $a = bx + c$, $\gcd(a, b) = \gcd(b, c)$.
- Proofs by contradiction:
  - $\sqrt{p}$ is irrational for p a prime (by contradiction).
  - The sum of a rational number and an irrational number is irrational (by contradiction).
- Floor and ceiling proofs using the definitions (as in Epp, 3rd Ed. §3.5 4th Ed §4.5).
- Know how to use a counterexample to <u>disprove</u> a universal statement.

V. Sequences and recursion (webwork type problems).

- Be able to use summation and product notation.
- Be able to use recursive formulas.
- Find the first several terms of a sequence given the inital terms and the recurrence formula.

VI. Know the formulas for the following sums:

- The sum of a geometric sequence.
- The sum of the first n integers.

VII. Know how to prove by induction!

- Be careful about the basic structure.
  - Use full sentences.
  - State the predicate.
  - Prove the base step.
  - State the assumption for the inductive step.
  - Do the inductive step.
- For a sequence defined recursively, given an explicit formula for the nth term, prove the formula is correct.

Relations

I. Functions and relations. Be able to do the following.

- Define terms!
  - Relation, inverse of a relation.
  - Function. Injective (one-to-one), surjective (onto) and bijective functions.
- Use a list of elements, an arrow diagram, a table (or graph), or a formula to define a function or relation.
- Determine whether a given relation is a function, or whether a given function is injective or surjective.
- Find the inverse relation of a function. Is it a function, injective, surjective?
- Give examples of functions satisfying various properties (see 7.2 #9 3rd and 4th Ed., 7.3 #5 2nd Ed.).
- Be able to compute the composition of two functions. See also problems §7.3 13-19 4th Ed., §7.4 #16-19 3rd Ed., §7.5 #15-18 2nd Ed.

II. Relations on a set. Be able to do the following.

- Define and be able to work with the following terms!
  - Reflexive, symmetric, transitive.
  - Equivalence relation
  - Partial order.
- Be able to work with the following terms.
  - Equivalence class.
  - Comparable, maximal, minimal, least, greatest.
- Verify or prove that a given relation R is symmetric (or reflexive, or transitive, or an equivalence relation, or a partial order).

- Verify that a given relation is irreflexive, or antisymmetric, or asymmetric (but I will give you the definition).
- Use tables, directed graphs, and lists of elements to represent a relation.
- For a relation R on A, be able to find the smallest relation containing R which is symmetric (ditto for reflexive, transitive, an equivalence relation, a partial order).
- Know the standard examples of equivalence relations (mod n, and exercises 8.3 # 20, 21, 25, 28-33 4th Ed., 10.3 #18, 19, 22, 23, 25, 27-29 3rd Ed.).
- Know the standard examples of partially ordered sets: $\leq$ for the integers (or rationals) divides on the integers; $\mathcal{P}(\mathcal{A})$ for a set A; D_n; (8.5 #16-21 4th Ed., 10.5 #16-21 3rd Ed.).
- Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

Counting

I. Know how to count!

- Know the inclusion-exclusion formula and be able to apply it and use a Venn diagram to illustrate.
- Know the 4 ways to choose and the formulas for 3 of them (I won't test "order unimportant, repetition allowed").
- Poker hands (I will describe the hand, and I may give you a strange deck or strange hand). See problems 9.5 #6-11, 4th Ed., 6.4 #6-11 3rd. Ed.