DISCRETE MATHEMATICS
Math 245

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Suggestions for preparing for the Second Exam

I. Things you should know about the integers and rational numbers:

- Be able to use (and recognize that you are using) commutativity, associativity, the additive and multiplicative identity, the additive inverse (and, for the rationals, the multiplicative inverse), distributivity.
- Be able to use (and recognize that you are using) properties of $<$. For example $a < b$ implies $a + c < b + c$.
- Be able to define prime, composite, divides, floor, ceiling.

II. Know the statements of the following theorems and know how to apply them (as in webworks problems):

- Quotient-remainder theorem.
- The unique factorization theorem.

III. Be able to do these computations.

- Use the Euclidean algorithm to find the greatest common divisor of two numbers.
- Convert an integer (base 10) into another base, and convert from any base into base 10.
- Add in any given base. Construct a multiplication table in a given base. Use a multiplication table to find a product of two numbers in any given base.
- Use unique factorization to solve equations involving integers.

IV. Know these standard proofs and proof methods.

- Divisibility results like:
  - Transitivity of divides.
  - If a divides b and a divides c then a divides $b + c$.
  - When $a = bx + c$, $\gcd(a, b) = \gcd(b, c)$.
- Proofs by contradiction:
  - There exist an infinite number of primes (by contradiction).
  - $\sqrt{p}$ is irrational for p a prime (by contradiction).
  - The sum of a rational number and an irrational number is irrational (by contradiction).
- Floor and ceiling proofs using the definitions (as in Epp, 3rd Ed. §3.5 4th Ed §4.5).
- Know how to use of a counterexample to <u>disprove</u> a universal statement.

V. Sequences and recursion (webwork type problems).

- Be able to use summation and product notation.
- Be able to use recursive formulas.
- Find the first several terms of a sequence given the inital terms and the recurrence formula.
- Find the formula for the nth term as a function of n for some simple examples.

VI. Know the formulas for the following sums:

- The sum of a geometric sequence.
- The sum of the first n integers.

VII. Know how to prove by induction!

- Be careful about the basic structure.
  - Use full sentences.
  - State the predicate.
  - Prove the base step.
  - State the assumption for the inductive step.
  - Do the inductive step.
- Types of induction proofs:
  - For a sequence defined recursively, given an explicit formula for the nth term, prove the formula is correct.
  - Prove divisibility results.
  - Use strong induction for a sequence defined by a recursion of order 2.