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Math 623: Matrix Analysis
Homework 7

Problem 1: Gershgorin disks. Do Bob Grone's three problems.

Problem from Horn 1.2.P7 Tridiagonal matrices.

(a) Use the formula for the characteristic polynomial of A in terms of the $E_k(A)$ to determine the characteristic polynomial of the tridiagonal matrices

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(b) Generalize to $n \times n$ tridiagonal matrices.

Problem from Horn 1.2.P22 A twist on circulant matrices. Let C_n be the $n \times n$ matrix with entry 1 in each $(i, i+1)$-position and in the $(n, 1)$-position and entry 0 elsewhere. Let $C_n(\epsilon)$ let be the matrix obtained from C_n by replacing the 1 in position $(n, 1)$ with $\epsilon > 0$.

(a) Show that the characteristic polynomial of $C_n(eps)$ is $t^n - \epsilon$.

(b) Find the spectrum and spectral radius of $C_n(\epsilon)$.

Problem from Horn 1.2.P23 If A is singular and has distinct eigenvalues, show that it has a nonsingular $(n-1) \times (n-1)$ principal minor.