Abstract Algebra
Math 521A
Michael E. O'Sullivan

Review for third exam

Rings and Ideals

- Know the definitions:
  - ring, commutative, identity, field;
  - unit, zero divisor, characteristic;
  - homomorphism, isomorphism.
  - ideal, principal ideal, generators of an ideal .
- Know how to:
  - Prove that a subset of a ring is an ideal (or show that it isn't).
  - Prove that a function is a homomorphism, or isomorphism (or show it isn't).
  - Show that two rings can't be isomorphic, because they have some different structure.
  - Identify the units and zero divisors in a ring.
- Know how to prove fundamental results about ideals in a commutative ring with identity.
  - The sum of ideals is an ideal.
  - The intersection of ideals is an ideal.
  - The kernel of a homomorphism is an ideal.
  - If I is an ideal in R and J is an ideal is S then $I \times J$ is an ideal in $R \times S$.
  - The annihilator of an ideal is an ideal.
- Know how to work with quotient rings.
  - If I is an ideal in R, the elements of R/I are written $a + I$ where $a \in R$.
  - $a + I = b + I$ when $a - b \in I$.
  - Addition in R/I is defined by $(a + I) + (b + I) = (a + b) + I$.
  - Multiplication in R/I is defined by $(a + I)(b + I) = (ab) + I$.
- Know these special examples.
  - Know what the ideals are in $\mathbb{Z}$, $\mathbb{Z}_n$, $F[x]$ and $F[x]/p(x)$ where F is a field.
  - Know how to find the simplest expression for an ideal in these rings.
  - Know some examples of non-principal ideals in $F[x, y]$ and $\mathbb{Z}[x]$.