

Abstract Algebra B
Math 521B
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Review for third exam (2010)

- Classification of finite abelian groups.
 - Be able to state and use the two main theorems:
The Fundamental Theorem of Finite Abelian Groups,
The Invariant Factor Theorem.
 - Be able to compute the elementary divisors and invariant factors of some given group.
 - §7.8 #14 §8.2 # 2,4,6,9a,10, 11.
- Sylow theorems
 - Know the definition of *p*-group and *Sylow p*-subgroup.
 - Be able to apply the Sylow theorems to some straightforward examples: §8.3 #1-7, and the groups of order 12 (see Thm 8.35).
- Be able to work with extension fields of \mathbb{Q} .
 - Use the rational roots test to check if a polynomial over \mathbb{Q} has a rational root.
 - Compute products and inverses.
- Know how to work with finite fields.
 - Know the Freshman's rule (see also Ex. 10.6# 10, 12).
 - Know the fundamental theorem for finite fields (as stated in class, but also spread out in Sec. 10.6)
 - * Any finite field is a simple extension of a field \mathbb{Z}_p (which we also write as \mathbb{F}_p).
 - * There exists a field of order p^n for each prime p and $n \in \mathbb{N}$.
 - * The field is unique up to isomorphism, since it is the splitting field of $x^{p^n} - x$.
 - * The multiplicative group of a finite field is cyclic.
 - Given an irreducible polynomial whose root α generates the multiplicative group of \mathbb{F}_{p^n} , construct the dictionary between powers of α and polynomials in α .
 - Use the dictionary to compute.