Homework 2

Turn in A(3), B(1,4,6), C(4), D(1,2) on Tuesday 2/7.
Problems on Varieties

(A) Varieties and non-varieties in $\mathbb{R}^n$:

(1) Do a few examples in [CLO] §1.2, e.g. #2, 4de, 5.
(2) Prove that a finite set of points is a variety. [CLO] §1.2 #6.
(3) (HW) Prove that the Cartesian product of varieties is a variety. [CLO] §1.2 #15d.

(B) Implicitly and parametrically defined varieties.

(1) (HW)[CLO]§1.2 #8 the punctured line is not a variety.
(2) [CLO]§1.3 #4 Implicitization of a parametrized curve.
(3) [CLO]§1.3 #5(b,c,d) Parametrizing the hyperbola.
(4) (HW) [CLO] §1.3 #6 Parametrizing the sphere.
(5) [CLO] §1.3 #11 parametrizing $x^2 - y^2 z + z^3$.
(6) (HW)[CLO] §1.4 #11, 12 The variety of a parametrized curve.

(C) Implicitization and parametrization

(1) [CLO] §2.1 #1d ideal membership in $k[x]$.
(2) [CLO] §2.1#2b Parametrize a variety described by linear equations.
(3) [CLO] §2.1#3b,c Find an implicit description of (the ideal of) a parametrized variety.
(4) (HW) [CLO] §2.1#5a-c Implicit formula for a parametrically defined curve in the plane.

(D) (HW) The Zariski topology. [These have short answers.]

(1) Show that the Zariski topology is indeed a topology.
(2) Show that the intersection of any two open sets is nonempty: hint [CLO] §1.1 Prop 5.

Definition: A **topology** on a set S, is a non-empty set of subsets of S, $\mathcal{O}$, called the open sets, such that the (1) the intersection of any two open sets is open, and (2) an arbitrary union (even an infinite union) of open sets is open. We define a **closed set** to be the complement of an open set: so the set of closed sets is $\mathcal{C} = \{O^c : O \in \mathcal{O}\}$.

Let k be an infinite field. The **Zariski topology** on k^n is defined by taking $\mathcal{C}$, the set of closed sets, to be the set of varieties.