Homework 8

Turn in a subset TBD on Thursday 4/16.

Problems on Ideals and Varieties

(A) Computations with ideals.

(1) Formulas for products of ideals: [CLO] 4.3#6.

(2) Formulas for products/sums/intersections and radicals [CLO] 4.3#7, 9, 12.

(3) Comaximality [CLO] 4.3#11.

(4) Linear ring homomorphisms [CLO] 4.3 #13, 14 .

(5) Let I and J be principal ideals in $k[\bar{x}]$, generated by $p(\bar{x})$ and $q(\bar{x})$. Unique factorization in $k[\bar{x}]$ gives a formula for a generator for $I \cap J$ from the factorizations of p and q (see [CLO] 4.3#2-5). The formula does not indicate how to actually compute a generator for $I \cap J$, for that we need [CLO] 4.3 Theorem 11 and elimination. Show in detail how this works: analyze $\Big(t\langle p\rangle + (1-t)\langle q\rangle\Big) \cap k[\bar{x}]$ and show that it is generated by $\mathrm{LCM}\,(p, q)$.

(B) Quotient ideals and associated geometry.

(1) Quotient of principal ideals [CLO] 4.4 #3 .

(2) Quotient ideal and geometry. [CLO] 4.4 #8.

(3) Quotient of a radical ideal is radical [CLO] 4.4#4.

(4) Quotient ideal and saturation [CLO] 4.4 #11,13.

(5) Quotient ideal and products [CLO] 4:4#16.

(C) Prime ideals correspond to irreducible varieties.

(1) Prime ideals and products/intersections [CLO] 4.5#3,4.

(2) Maximal ideals in non-algebraically closed fields [CLO] 4.5#8,9.

(3) Irreducible polynomials and prime ideals [CLO] 4.5#11.

(4) A characterization of radical ideals [CLO] 4.5#12.