Math 621: Advanced Topics in Algebra
Commutative Algebra and Algebraic Geometry
Spring 2022 Syllabus

Schedule: Tu. Th. 12:30-1:45 in GMCS 405 starting Tu. Jan. 21.
Date of Final: Th. May 15, 10:30-12:30, GMCS 405.

Instructor: Michael E. O'Sullivan (he).
You may call me Mike or something more formal if you prefer.
Office Hours: GMCS 582. TBD. Please fill out this poll the first week of classes. `https://whenisgood.net/ahspakf`
Students are encouraged to email for an appointment at other times.
Email: mosullivan@sdsu.edu

Websites and Communications: Please see my website for detailed information about the course. My "Teaching" page has a link to the course page, which has lots of information and links to assignments and to a schedule of topics.

My home page	`https://mosullivan.sdsu.edu`
My Teaching page	`https://mosullivan.sdsu.edu/teaching.html`
My email address	`https://mosullivan@sdsu.edu`

I will sometimes use Canvas to communicate to the whole class, to post grades, and for miscellaneous other things. To contact me it is best to use the email address above.

It is extremely important to me that all students feel welcome and supported in the class. Please let me know about the name and pronouns you use. Feel free to communicate with me about any concerns you have with the class environment.

Course Description: We will study algebraic geometry, one of the oldest and richest areas of mathematics. During the 20th century, the theoretical and very abstract side of the subject was prominent, but, with the availability of computers, the computational roots have been reinvigorated. There has also been a lot of effort toward making the subject more accessible. This course will develop the theory that ties geometrical concepts to algebraic methods and will address some computational tools.

What is algebraic geometry? Think back to high-school algebra where you graphed polynomial equations and perhaps found the intersection of plane curves such as a line and a parabola or more general curves defined by polynomials. Now think about higher dimensional space and consider intersections of hyper-surfaces defined by polynomial equations. Such objects are called algebraic sets or algebraic varieties. What is the dimension? How many components are there? What is the simplest way to describe the intersection? These are some of the geometric questions arising in algebraic geometry.

The fundamental result in algebraic geometry is the algebra-geometry "dictionary" which gives a precise relationship between geometrical objects and algebraic ones: between varieties in n-dimensional space and radical ideals in the polynomial ring in n variables. Algebra provides tools for formalizing and being precise about geometric concepts, which can be rather intuitive. Conversely, algebraic results have a geometric interpretation that brings richness to abstract formulas.

The fundamental tools in computational algebraic geometry are Gröbner bases for ideals and Buchberger's algorithm to compute them. Groebner bases are a generalization of the greatest common divisor of integers. Just as the Euclidean algorithm may be used to compute the gcd, Buchberger's algorithm is used to compute a Gröbner basis for an ideal.

In the last few decades, numerous applications of algebraic geometry have been discovered: in coding theory, cryptography, robotics, object recognition, engineering, genomics etc. Some links that show the scope of recent work are: The Society for Industrial and Applied Mathematics Activity Group on Algebraic Geometry; and its conference page; and the work of Bernd Sturmfels. Powerful computational software has also been developed. See for example SageMath (which can include Macaulay 2 and Singular), and Magma. These computational tools are of great importance in applications.

Course Materials.

- E. Clader, D. Ross, *Beginning in Algebraic Geometry* Available here. I **encourage** you to get a physical copy. I will have it printed and spiral bound for about $35.
- Cox, Little O'Shea: *Ideals, Varieties, and Algorithms* (2nd, 3rd or 4th Edition). This is the first book on computational algebraic geometry designed for a broad audience. Not required but useful.
- O'Sullivan: *Groups, Rings, and Fields* Available here Course Notes for Math 620 as background material.
- O'Sullivan: Supplementary lecture notes delivered as the semester progresses.
- William A. Stein et al. Sage Mathematics Software The Sage Development Team, 2011, `https:www.sagemath.org`
- The Sage Reference Manual `https://doc.sagemath.org/html/en/reference/index.html`
- SDSU SageMath Tutorial `https://mosullivan.sdsu.edu/Teaching/sdsu-sage-tutorial/index.html`
- Magma online calculator. `http://magma.maths.usyd.edu.au/calc/`

I may include some computational work with software in the course; we'll see how that develops. It is also an excellent option for student projects. Magma is a package for all kinds of algebraic computation that is very stable and of high quality. The documentation is a bit heavy, but I have some code to get you started. SageMath is an open source mathematics software package modeled in part on Magma. It incorporates numerous other open-source packages into a unified package. The SDSU Sage tutorial, written by David Monarres and me with updates from Matteo Polimeno, will help you get started.

Prerequisites: Experience with abstract algebra: ring theory, basic field theory, vector spaces, basic group theory. Of particular importance is commutative ring theory. My course notes [Secs. 1.1, 1.3; Ch. 4; Ch. 5 (§1-3)] are more than sufficient.

Format: Class time will mix lecture with problem solving. We may also spend some time using SageMath or Magma in the computer lab.

Learning Outcomes: It is standard these days to have learning outcomes for every course; rather than simply listing the topics covered. My approach to this is as follows. In every math course that I teach, I want students to advance in the skills listed below (adapted from the Degree Learning Outcomes for the SDSU math major as presented on the department website). In this course, we do this work in the context of commutative algebra and algebraic geometry.

Foundational knowledge. State major definitions, axioms, and theorems and use examples to illustrate.

Use logical reasoning. Read a proof and explain the logic and derivations. Write a mathematical proof using an appropriate method.

Use algebraic tools and methods. Derive answers, apply algorithms, and compute, both by hand and using mathematics software.

Explore mathematical ideas independently. Have confidence to read challenging material that is beyond that explored in a textbook or class.

Communicate mathematical ideas effectively. Make progress toward the mathematicians goal: writing that gets to the essence of the matter and is brief, clear, and polished.

Nurture the learning of others. Work with others in a way that is collegial, inclusive and empowering. Contribute, but seek understanding of other perspectives.

Grading: We will have homework assignments with proofs and computational exercises. I may incorporate some straightforward computer assignments as well.

There will be two midterms and I will give you a pretty clear idea of the content for tests beforehand.

There will be a final project, with a great deal of latitude in choice of topic. You may focus on theoretical questions, implementation of an algorithm, an applied problem, or some combination. I have plenty of references, including recent research, that should be accessible to you by the end of the course. You may also develop an educational module such as something for advanced high-school students. More information about the project will be provided later in the semester. The project will include a class presentation in the last week of class, or on the day of the final.

The final grade will be apportioned as indicated in the table ± 50 points for each item.

Weekly work	450
Midterm Tests (2)	300
Final Project	250
Total	1000

Grading Scale: A: 100-85%, B: 84-70%, C: 69-55%, D: 55-40%, F: below 40%. The lower 5 points of each range may be awarded a $-$ and the upper 5 may be awarded a $+$ based on the instructor's judgement of a student's cumulative progress in the course.

Collaboration Policy: You are encouraged to work together to solve problems, but you should write the solutions individually. Significant levels of collaboration on the solution of a problem should be noted on any homework.

On homework and tests, your solutions should be understandable by a peer. Not every detail has to be explained, provided a peer would know how to fill in the details. This is the art of exposition, knowing your audience and how to succinctly communicate essentials.

Essential Student Information: For essential information about student academic success, please see the SDSU Student Academic Success Handbook.

SDSU provides disability-related accommodations via the Student Ability Success Center (sascinfo@sdsu.edu — `http://sdsu.edu/sasc`). Please allow 10-14 business days for this process.

Land Acknowledgement For millennia, the Kumeyaay people have been a part of this land. This land has nourished, healed, protected and embraced them for many generations in a relationship of balance and harmony. As members of the San Diego State University community, we acknowledge this legacy. We promote this balance and harmony. We find inspiration from this land, the land of the Kumeyaay.