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Math 627A: Modern Algebra I

Homework I

Problem 1: Use the Euclidean algorithm to express the greatest common divisor as a linear combination of the given

- 89, 24
- 24, 10, 12
- $f = x^6 + 1$ and $g = x^4 + x^3 + x^2 + 1$ as elements of $\mathbb{F}_2[x]$.

Problem 2: Use the result that $\gcd(a, b)$ is a linear combination of a and b to prove that $\gcd(a, b, c) = \gcd(a, \gcd(b, c))$.

Problem 3: Write a multiplication table for $\mathbb{F}_3[x]/\langle x^2 + x + 2\rangle$. [You may omit 0. It may be easier to take the elements in the order $1, x, x + 1, x + 2$ followed by twice each.]

Problem 4: Use a linear system to find the inverse of $x + 3$ in $\mathbb{Q}[x]/\langle x^2 + 2\rangle$.

Problem 5: Let σ be the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 5 & 4 & 1 & 2 \end{pmatrix}$.

- Write σ in cycle notation.
- Compute σ^2.
- Compute σ^{-1}
- Find the order of σ.

Problem 6: Let $(U_n, *)$ be the group of invertible elements of $\mathbb{Z}_n$. Find all n such that $(U_n, *)$ is isomorphic to

- $(\mathbb{Z}_2, +)$.
- $(\mathbb{Z}_4, +)$.
- $(\mathbb{Z}_2 \times \mathbb{Z}_2, +)$.

Problem 7: Define a *hemigroup* to be a set G with an operation $*$ that is associative, has an identity element, and such that each element has a *right* inverse. Show that the right of a is also a left inverse of a, so that a hemigroup is actually a group.