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Math 627A: Modern Algebra I

First Exam

Please read the following problems and their solutions in Ash's text. I've grouped (no pun intended) problems that are related.

- §5.1 pr. 1,2,3,8,9; 4,5; 6,7.
- §5.2 pr. 3; 4,5.
- §5.4 pr. 1; 2,3; 5,6; 7.
- §5.5 pr. 1; 3; 4.
- §5.6 pr. 1; 3; 4-9.
- §5.7 pr. 3,7,8.

The exam will mainly be drawn from these problems.
You should also review the classification of abelian groups (statements of theorems not the proofs) and our discussion of groups of order 8.

Homework IV

due: Fri. 11/31/08

Clarity of exposition is crucial in this assignment. You work should be understandable to a fellow student. You may work together to solve problems, but your solutions should be written independently.

Problem 1: Let G be a group with p^r elements.
(a) Show that G has a subgroup of any possible order, p^i for $i = 0, 1, 2, \ldots, r$.
(b) Show that G is solvable.

Problem 2: Let G be a group of order $p^r q$. Fiddle with the proof in Ash concerning groups of order $p^2 q$ to establish the following.
(a) If $p > q$ then G is solvable.
(b) If $p < q$ and $p^i \not\equiv 1 \mod q$ for $i = 1, \ldots, r-1$ then G is solvable.

Problem 3: Show that all groups of order less than 60 are solvable. Use the two previous problems and 5.5#6 to whittle down the list to a few special cases. Then attack the special cases using problem 5.1#9 and 5.4#1.