Mike O'Sullivan
Department of Mathematics
San Diego State University

Spring 2009

Math 627B: Modern Algebra II

Homework I

Problem 1: The radical of a polynomial. IVA §1.5 #14, 15.

Problem 2: The variety of several polnomials in $\mathbb{C}[x]$. IVA §1.5 #16.

Problem 3: Sketching some varieties in $\mathbb{R}^n$: IVA §1.2 #2, 4e, 5.

Problem 4: Varieties and non-varieties.
(a) Prove that a finite set of points is a variety. §1.2 #6.
(b) Prove that the punctured line is not a variety. §1/2 #8.

Problem 3: The Euclidean Algorithm
Input $f, h \in F[x]$.
Output $r, u, v, y, z \in F[x]$ such that (1) $uf + vh = r = \gcd(f, h)$, and
(2) $yf = -zh = \operatorname{lcm}(f, h)$.
Initialize

$$M = \begin{bmatrix} r & u & v \\ s & y & z \end{bmatrix} = \begin{bmatrix} f & 1 & 0 \\ h & 0 & 1 \end{bmatrix}$$

Algorithm While $s \neq 0$ do
$q \leftarrow$ quotient (r, s)
$M \leftarrow \begin{bmatrix} -q & 1 \\ 1 & 0 \end{bmatrix} M$

(a) Prove that at every iteration

$$\begin{bmatrix} u & v \\ y & z \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix}$$

(b) Show that at termination u, v give the $\gcd(f, h)$ and y, z give the $\operatorname{lcm}(f, h)$ as claimed in the output statement. [Hint: Show that $uz - yv = \pm 1$ at every iteration.]