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Math 627B: Modern Algebra II
Homework I

Problems 1 and 2: Due Th. 3/25 2012.

Problem 1: The dicyclic group Di_n is defined by the following generators and relations:

$$\langle a, d \mid a^{2n} = e, d^2 = a^n, da = a^{-1}d \rangle$$

- **(a)** Show that Di_n has $4n$ elements, by showing that each element can be represented in the form $a^i d^j$ (and derive a uniqueness result relative to this form). Find the product of two arbitrary elements, written in this form.
- **(b)** For n prime, compute the order of each element of Di_n, using the form in (a). Make a table for Di_n showing (1) each order that is possible, (2) the number of elements of that order, (3) the elements of that order.
- **(c)** Use part (b) to show that Di_3 is not isomorphic to D_6. (Use sage as an aid, but you will have to work with the representation from part (a)).
- **(d)** Show that there is a surjective homomorphism from Di_n to the dihedral group D_n. What is the kernel?
- **(e)** Show that Di_2 is isomorphic to the quaternions.
- **(f)** Use sage, and your wits, to draw the subgroup lattice diagram for Di_3. Remark on the correspondence theorem relative to part (d).
- **(g)** Show that Di_3 is a semidirect product of two smaller groups.

Problem 2: Shorties.

- **(a)** Show that the intersection of two normal subgroups of G is normal in G.
- **(b)** If every element in G has order 2 show that G is ableian.
- **(c)** If G has even order then G has an element of order 2.
  (Consider the pairing of g with g^{-1}).

Problem 3: A group is *metabelian* when it has a normal subgroup N such that N and G/N are both abelian. A group is *metacyclic* when it has a normal subgroup N such that N and G/N are both cyclic.

- **(a)** Show that S_3 is metacyclic.
- **(b)** Show that A_4 is metabelian but not metacyclic.
- **(c)** Prove that any subgroup of a metabelian group is also metabelian.
- **(d)** Prove that any quotient group of a metabelian group is metabelian. [Look carefully at the proof of the 2nd isomorphism theorem and adapt it to this question.]