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Math 627B: Modern Algebra II
Homework VII

Due Th. 4/19 2012.

Problem 1: Minimal polynomials for elements of $\mathbb{Q}(\sqrt[3]{2})$ (see [A] 3.1#4,5).

(a) Find the minimum polynomial of $\sqrt[3]{2}+1$ over $\mathbb{Q}$. [There is a clever way that involves little computation, but don't feel obliged to use it!]

(b) Find the minimum polynomial of $\sqrt[3]{2}+\sqrt[3]{4}$ over $\mathbb{Q}$.

Problem 2: Sixth roots of unity.

(a) Factor x^6-1 completely over $\mathbb{Q}$.

(b) Show that the splitting field of x^6-1 is of degree 2 over $\mathbb{Q}$.

Problem 3: Let E be the splitting field of x^6-2.

(a) Show that x^6-2 is irreducible over $\mathbb{Q}$.

(b) Analyze the splitting field E of x^6-2 over $\mathbb{Q}$ in four ways. In each case, (1) refactor x^6-2 over any intermediate fields and (2) compute the dimensions of the intermediate fields over $\mathbb{Q}$.

- Adjoin $\sqrt[6]{2}$ then adjoin a root of a factor of x^6-2.
- Adjoin a sixth root of unity, then split x^6-2.
- Adjoin $\sqrt{2}$ and refactor x^6-2. Then split the factors.
- First construct the splitting field, K, of x^3-2 in two steps. For each step, refactor x^6-2. Finally split the irreducible factors of x^6-2 over K.

(c) Compute $[E:\mathbb{Q}]$

(d) Identify all of the subfields of E that you encountered.