

# DISCRETE MATHEMATICS

## Math 245

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### Suggestions for preparing for the Second Exam

I. Things you should know about the integers and rational numbers:

- Be able to use (and recognize that you are using) commutativity, associativity, the additive and multiplicative identity, the additive inverse (and, for the rationals, the multiplicative inverse), distributivity.
- Be able to use (and recognize that you are using ) properties of  $<$ . For example  $a < b$  implies  $a + c < b + c$ .
- Be able to define prime, composite, divides, floor, ceiling.

II. Know the statements of the following theorems and know how to apply them (as in webworks problems):

- Quotient-remainder theorem.
- The unique factorization theorem.

III. Be able to do these computations.

- Use the Euclidean algorithm to find the greatest common divisor of two numbers.
- Convert an integer (base 10) into another base, and convert from any base into base 10.
- Add in any given base. Construct a multiplication table in a given base. Use a multiplication table to find a product of two numbers in any given base.
- Use unique factorization to solve equations involving integers.

IV. Know these standard proofs and proof methods.

- Divisibility results like:
  - Transitivity of divides.
  - If  $a$  divides  $b$  and  $a$  divides  $c$  then  $a$  divides  $b + c$ .
  - When  $a = bx + c$ ,  $\gcd(a, b) = \gcd(b, c)$ .
- Proofs by contradiction:
  - There exist an infinite number of primes (by contradiction).
  - $\sqrt{p}$  is irrational for  $p$  a prime (by contradiction).
  - The sum of a rational number and an irrational number is irrational (by contradiction).
- Floor and ceiling proofs using the definitions (as in Epp, §3.5).
- Know how to use of a counterexample to disprove a universal statement.

V. Sequences and recursion.

- Be able to use summation and product notation.
- Be able to use recursive formulas.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.
- Find the formula for the  $n$ th term as a function of  $n$  for some simple examples.

VI. Know the formulas for the following sums:

- The sum of a geometric sequence.
- The sum of the first  $n$  integers.

VII. Know how to prove by induction!

- Use full sentences.
- State the predicate.
- Prove the base step.
- State the assumption for the inductive step.
- Do the inductive step.