

NAME:

DISCRETE MATHEMATICS

Math 245

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Quiz

Thursday, September 13, 2012

Do all problems. Show your work!

I. [20 pts.] Consider the logical statements $p \implies (q \vee r)$ and $(p \wedge \sim q) \implies r$.

(a) Use a truth table to show that the two statements are logically equivalent.

Write a sentence of explanation.

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$p \wedge \sim q$	$(p \wedge \sim q) \rightarrow r$
T	T	T	T	T	F	T
T	T	F	T	T	F	T
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	F	T
F	F	F	F	F	F	T

$\underbrace{\hspace{10em}}_{\textcircled{1}} \qquad \qquad \qquad \underbrace{\hspace{10em}}_{\textcircled{2}}$

The two statements are equivalent as the associated columns ① and ② are equal.

(b) Using the equivalence above, rewrite the sentence "If I graduate then I will become a doctor or a lawyer." in an equivalent form.

If I graduate and I don't become a doctor then I will become a lawyer.

II. [10 pts.] Simplify completely using the logical equivalences that we have established:

$$(p \wedge (q \vee r)) \vee (p \wedge \sim q)$$

$$\begin{aligned} (p \wedge (q \vee r)) \vee (p \wedge \sim q) &\equiv \\ p \wedge ((q \vee r) \vee (\sim q)) &\equiv \\ p \wedge (q \vee \sim q \vee r) &\equiv \\ p \wedge (t \vee r) &\equiv \\ p \wedge t &\equiv \\ p & \end{aligned}$$

III. [10 pts.] Write a circuit that exactly implements the following statement. Do not simplify.

$$(p \wedge (q \vee \sim r)) \vee (\sim p \wedge \sim q)$$

