Math 620: Groups, Rings, and Fields
Fall 2024 Syllabus

Schedule: Monday, Wednesday, Friday, 9:00 in GMCS 307.
First day of class: Monday, Aug. 26
Last day of class: Wednesday, Dec. 11.
Holidays: Monday Sept. 2; Monday, Nov. 11; Wednesday-Friday, Nov. 22-24.
Test 1: (roughly) Monday, Oct. 7.
Test 2: The week of Nov. 4-8.
Final Test: Monday Dec. 16, 8:00-10:00.

Instructor: Michael E. O'Sullivan (he).
You may call me Mike or something more formal if you prefer.
Office Hours: GMCS 582 Monday, Wednesday, Friday 10:00-11:00.
Other times TBD; also by appointment.
I enjoy meeting with students, so don't hesitate to ask.
Email: mosullivan@sdsu.edu
Website: mosullivan.sdsu.edu

Websites and Communications: Please see my website for detailed information about the course. My "Teaching" page has a link to the course page, which has lots of information and links to assignments and to a schedule of topics.

I will set up a Discord server for the class. There is an initial invitation that just gets you access to the class. When you first join, you will not be able to post. Send me a Discord message @mosullivan-math with your full name (first & last) and RedID number, and I will change your role to @students, giving you posting permission.

Please feel free to post questions and to answer questions. Relative to HW problems, provide guidance not solutions. As with all interactions in the course: Nurture the learning of others. Work with others in a way that is collegial, inclusive and empowering. Contribute, but seek understanding of other perspectives.

I will sometimes use Canvas to communicate to the whole class, to post grades, and for miscellaneous other things. To contact me it is best to use the email address above.

It is extremely important to me that all students feel welcome and supported in the class. Please let me know about the name and pronouns you use. Feel free to communicate with me about any concerns you have with the class environment.

Course Description: We will study the core objects in abstract algebra: groups, rings and fields. The term "abstract" highlights the approach to mathematics that emerged in the 20th century in which there was less reference to "real objects" and a greater emphasis on proving theorems from a small set of simple axioms. For example, the symmetries of a 3-dimensional object tell you something about the nature of the object.

But, symmetry itself is interesting. Symmetries of an object can be composed (apply one then another) and produce another symmetry. Each symmetry has an inverse, which undoes the given symmetry. Abstractly, one can consider the properties of a set (e.g. symmetries) with an operation (e.g. composition) and an inverse to the operation. We call such an object a *group*. Discovering the structure and properties of abstracts groups was a massive project in the last century.

Ring theory evolves from recognizing that the basic properties of the integers (from grade school!)—two operations $(+, *)$, with commutativity, associativity, distributivity—extend to polynomials and can be extended further to abstractly defined objects. Similarly, the properties of the rational numbers (now every nonzero element has a multiplicative inverse) can be considered in an abstract axiomatic way. This is the area called *field* theory.

I love this material because it gives a deeper and richer understanding of school mathematics, but also unveils a world of very complex and intricate beauty.

Course Materials. I am writing lecture notes for the course, and will distribute them as I finalize them. You might want a physical copy of the Hungerford book (a cheap, 2nd edition) as a reference with more detail than my notes. Dummit & Foote's book is a great resource if you plan to continue studies in algebra.

- Thomas Hungerford: *Abstract Algebra: An Introduction*. This is the textbook we use for our undergraduate algebra courses. It is well written and should be a nice reference to flesh out details and give examples.
- David Dummit, Richard Foote, *Abstract Algebra*. A massive standard reference for graduate level algebra, full of examples and detailed proofs.
- Robert Ash: *Abstract Algebra: The Basic Graduate Year*. Available for free at `https://faculty.math.illinois.edu/~r-ash/Algebra.html`. Physical copies are published by Dover and are cheap.
- William A. Stein et al. Sage Mathematics Software The Sage Development Team, 2011, `https:www.sagemath.org`
- SDSU SageMath Tutorials
  `https://mosullivan.sdsu.edu/Teaching/sdsu-sage-tutorial/index.html`
  `https://doc.sagemath.org/html/en/thematic_tutorials/index.html`
- Magma online calculator. `http://magma.maths.usyd.edu.au/calc/`

I'd like to include some computational work with software in the course; we'll see how that develops. Magma is a package for all kinds of algebraic computation that is very stable and of high quality. The documentation is a bit heavy, but I have some code to get you started. SageMath is an open source mathematics software package modeled in part

on Magma. It incorporates numerous other open-source packages into a unified package. The SDSU Sage tutorial, written by David Monarres and me with updates from Matteo Polimeno, will help you get started. There are also numerous tutorials at the SageMath site.

Prerequisites: Some experience with abstract algebra: group theory, ring theory, and number theory. If you have just studied one of these topics, or did not obtain a grade of B or better, be prepared to work a bit harder (allot time!) in the course.

Format: Class time will mix lecture with problem solving. We may also spend some time using SageMath or Magma in the computer lab. I will assign specific pages from my notes to be read before class. You may not understand some of the material, but, read the assigned pages, formulate questions that you have, and be prepared to discuss this in class. I have some short ($5-15$ min.) recorded lectures on some topics, which will free class time for discussion of problems. Be prepared to present your work in class and also to work on problems in class.

Learning Outcomes:. It is standard these days to have learning outcomes for every course; rather than simply listing the topics covered. My approach to this is as follows. In every math course that I teach, I want students to advance in the skills listed below (adapted from the Degree Learning Outcomes for the SDSU math major as presented on the department website). In this course we do this work in the context of groups, rings and fields.

Foundational knowledge. State major definitions, axioms, and theorems and use examples to illustrate.

Use logical reasoning. Read a proof and explain the logic and derivations. Write a mathematical proof using an appropriate method.

Use algebraic tools and methods. Derive answers, apply algorithms, and compute, both by hand and using mathematics software.

Explore mathematical ideas independently. Have confidence to read challenging material that is beyond that explored in a textbook or class.

Communicate mathematical ideas effectively. Make progress toward the mathematicians goal: writing that gets to the essence of the matter and is brief, clear, and polished.

Nurture the learning of others. Work with others in a way that is collegial, inclusive and empowering. Contribute, but seek understanding of other perspectives.

Grading: We will have homework assignments with proofs and computational exercises. I may incorporate some straightforward computer assignments as well. You are encouraged to work with one another to solve the problems on the homework, but solutions should be written individually.

There will be three tests: two midterms and a final. I will give you a pretty clear idea of the content of each beforehand.

The final grade will be apportioned as indicated in the table +/- 20% of the points for each item.

Weekly work	400
Miscellaneous	50
Midterm Test 1	150
Midterm Test 1	150
Final Test	250
Total	1000

Grading Scale: A: 100-85%, B: 84-70%, C: 69-60%, D: 59-50%, F: below 50%. The lower portion of each range may receive a $-$.

Collaboration Policy: You are encouraged to work together to solve problems, but you should write the solutions individually. Significant levels of collaboration on the solution of a problem should be noted on any homework.

On homework and tests, *your solutions should be understandable by a peer*. Not every detail has to be explained, provided a peer would know how to fill in the details. This is the art of exposition, knowing your audience and how to succinctly communicate essentials.

Essential Student Information: For essential information about student academic success, please see the SDSU Student Academic Success Handbook.

SDSU provides disability-related accommodations via the Student Ability Success Center (sascinfo@sdsu.edu — `http://sdsu.edu/sasc`). Please allow 10-14 business days for this process.

Land Acknowledgement For millennia, the Kumeyaay people have been a part of this land. This land has nourished, healed, protected and embraced them for many generations in a relationship of balance and harmony. As members of the San Diego State University community, we acknowledge this legacy. We promote this balance and harmony. We find inspiration from this land, the land of the Kumeyaay.