Information Theory

1) (from PRP) We are given two urns, each containing a collection of colored balls. Urn I contains two white and three blue balls, whilst urn II contains three white and four blue balls. A ball is drawn at random from urn I and put into urn II, and then a ball is picked at random from urn II and examined.

What is the probability that it is blue?

Given that it is blue, what is the probability that the ball drawn from urn I was blue?

2) (from PRP) A man possesses 5 coins, 2 are double headed, 1 is double tailed and 2 are normal. He shuts his eyes, picks a coin at random and tosses it. What is the probability that the lower face of the coin is a head?

He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

He shuts his eyes again, and tosses the coin. What is the probability that the lower face is a head?

He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

He discards this coin, picks one of the remaining coins at random and tosses it. What is the probability that the lower face is a head?

3) (Let's Make a Deal) There are three doors, and behind one is a shiny new bicycle, while the other two each have a old broken cart. You have one chance to choose the correct door. You choose number 2. The game host, in a genial mood, shows you that behind door number one is an old cart, and he gives you the opportunity to change your mind. Should you?

4) (from PRP) Urn A contains 11 red and 7 green balls, urn B contains 4 red and 1 green ball, and urn C contains 2 red and 6 green balls. The three urns are placed in a dark room. Someone stumbles in, gropes around, finds an urn and draws a ball from it.

What is the probability that the ball is red?

What is the proportion of red balls in the room?

If the ball is green, what is the probability that it was drawn from urn A?

5) (Markov Chains) Let $X = Y = Z = \{0, 1\}$.

Find a distribution on $X \times Y \times Z$ which is not a Markov chain.

How many degrees of freedom are eliminated by imposing the constraint that the distribution be a Markov chain?

If P is a distribution on $X \times Y \times Z$ which can be expressed as a Markov chain under the ordering X, Y, Z is it also a Markov chain with respect to other orderings—e.g. X, Z, Y; Y, X, Z; etc.? Prove or give counter-examples (or both).

6) Construct an example illustrating Simpson's paradox.

Let A, B, C be three random variables taking two values each. Choose the number of individuals in each of the eight possible categories (choose easy round numbers) so that 1) $P_B(A)$ is *much* larger than $P_{B^c}(A)$, but 2) additional conditioning with respect to either C (or C^c) gives equal probabilities.

Create a reasonable scenario in which one might get these measurements, and interpret them.

7) EPT Ch. 5 #20.