Linear Algebra
Math 254
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Review for first exam
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Solving Linear Systems. Know how to:

- Transform a system of linear equations into a matrix equation.
- Transform a traffic flow problem into a linear system, and then into a matrix equation.
- Solve a system using Gaussian elimination.
- Explain the steps that you use (row replacement steps and row exchange steps).
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- Write a vector x as a sum of vectors $v_1, \ldots, v_m$ (or check that it can't be done) using Gaussian elimination.
- Check whether vectors $v_1, \ldots, v_n$ are linearly independent using Gaussian elimination.
- Invert a matrix using Gaussian elimination.

The Language of Linear Transformations. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be the linear transformation given by the $m \times n$ matrix A. Let $b \in \mathbb{R}^n$.

- Be able to translate tranformations on $\mathbb{R}^2$, stated as rotations, reflections, or dilations/contractions into matrix form. Be able to work with diagrams of the image of a transformation $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ (§1.9#1-14).
- The set of x satisfying $T(x) = b$ is the same set as the set of x satisfying $Ax = b$. be able to translate back and forth between the two: (§1.9#17-28).
- **Theorem 6:** Let p be one solution to $Ax = b$. The solution set of $Ax = b$ is all $p + v_h$ where v_h varies among the solutions to $Ax = 0$.
- **Theorem 11:** T is a one-to one function if and only if the only solution to $Ax = 0$ is the zero vector in $\mathbb{R}^n$. [This follows directly from Theorem 6. When there is only one homogeneous solution, there is at most one solution to $Ax = b$.]
- There are many ways to express the same essential idea. In the following table, as you read across each row you see different ways of saying a particular concept. The final line contains most of the Invertible Matrix Theorem (Theorem 8 in Chapter 2).

Table 1: Equivalent properties for an $m \times n$ matrix A

$Ax = b$ has a solution for all $b \in \mathbb{R}^m$	RREF of A has a pivot in each row	The columns of A span $\mathbb{R}^m$	The linear transformation $x \longmapsto Ax$ is *onto*	There is an $m \times m$ matrix Z such that $ZA = I_m$	Note: $n \geq m$
$Ax = 0$ has one solution, $x = 0$	RREF of A has a pivot in each column	The columns of A are linearly independent in $\mathbb{R}^m$	The linear transformation $x \longmapsto Ax$ is *one-to-one*	There is an $n \times n$ matrix Z such that $AZ = I_n$	Note: $n \leq m$
$Ax = b$ has exactly one solution for each $b \in \mathbb{R}^m$	RREF of A has a pivot in each row and each column	The columns of A are linearly independent and span $\mathbb{R}^m$	The linear transformation $x \longmapsto Ax$ is *one-to-one* and it onto	there is a $n \times n$ matrix Z such that $AZ = ZA = I_n$	Note: $n = m$