Def A function, f, from X to Y (sets) is a rule that assigns to each $x \in X$ exactly one element of Y. We call it $f(x)$

Write $f : X \longrightarrow Y$
$$x \longmapsto f(x)$$

Other terms: map, transformation, operation

Def f is surjective when every element of Y has a preimage,
$$\forall y \in Y, \ \exists x \in X \text{ such that } f(x) = y$$

f is injective when any two distinct elements of X have different images
$$\forall x, x' \in X, \text{ if } x \neq x' \text{ then } f(x) \neq f(x')$$

Def A vector space over F is a set with two functions (often called operations)
$$V \times V \xrightarrow{+} V$$
$$(v, w) \longmapsto v + w$$
and
$$F \times V \longrightarrow V$$
$$(a, v) \longmapsto av$$
satisfying

$+$ is commutative, associative has an identity, $\overline{0}$, admits inverses $v \longleftrightarrow -v$

$*$ is associative, $1v = v$ and $*$ distributes over $+$.

Thm Let V be a finite dimensional* vector space over F.

(1) V has a basis

(2) Any two bases have the same length, which is called $\dim(V)$.

(3) For U a subspace of V

(a) $\dim(U) \leq \dim V$ with equality iff $U = V$.

(b) there is a subspace W such that $U \oplus W = V$.

* It has a spanning list