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Math 623: Matrix Analysis
Review for First Test

Gaussian elimination

- Know the elementary matrices.
- Explain why every invertible matrix is the product of elementary matrices.
- Be able to state and use the rank-nullity theorem.
- Be able to state and use the theorem concerning $PLDU$ factorization.

Orthogonality

- Know the definition of inner product spaces, orthogonal matrix and unitary matrices.
- Know the Gram-Schmidt process and the relationship with QR factorization.
- For $A \in \mathcal{M}_n(\mathbb{R})$ there are several properties that are equivalent to A being orthogonal: orthogonal rows, orthonormal columns, length preserving, $xAy = xy$. Be able to prove these equivalences.

Abstract vector spaces and linear tranformations

- Be able to state and work with the definitions for vector space, linear transformation, invariant space.
- Understand the relationship between similarity of matrices and change of basis.

Jordan form over $\mathbb{C}$ and Jordan form over $\mathbb{R}$.

- We will use the notation $J_k(i)$ for a $a \times a$ Jordan block with a on the diagonal.
- You may use the notation $C(a,b)$ for a $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.
- We will use the notation $A \oplus B$ for the block diagonal matrix $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.
- Classify all possible Jordan forms for nilpotent matrices of a given size (not too big).
- Classify all possible Jordan forms for a given characteristic and minimal polynomial (real and complex case).
- Give the Jordan form of J_n^k.
- Be able to state the main theorem on Jordan form and understand its interpretation in terms of similarity and of generalized eigenspaces of a linear transformation.
- Be able to state and use the theorem about Jordan form over $\mathbb{R}$.