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Math 623: Matrix Analysis
Homework 2

Problem 1: Factor A into the form $A = PLDU$, with P a permutation matrix, L lower triangular with 1 on the diagonal, D diagonal, and U upper echelon.

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 2 & 7 \\ 3 & 6 & 4 & 8 \\ -2 & -4 & -1 & -4 \end{bmatrix}$$

Problem 2: Some subspaces.

(a) Show that the nullspace of a matrix $A \in \mathcal{M}_{m,n}$ is a subspace of $\mathbb{R}^n$.

(b) Suppose that V is the direct sum of subspaces $U_1, \ldots, U_t$: $V = U_1 \oplus U_2 \oplus \cdots \oplus U_t$ Show that $\dim V = \sum_{i=1}^{t} \dim U_i$.

Problem 3: Let $\mathcal{U} = \begin{bmatrix} u_1 & u_2 & \ldots & u_n \end{bmatrix}$, $\mathcal{W} = \begin{bmatrix} w_1 & w_2 & \ldots & w_n \end{bmatrix}$ and $\mathcal{Y} = \begin{bmatrix} y_1 & y_2 & \ldots & y_n \end{bmatrix}$ be bases for vector space V. Let S be the change of basis matrix from $\mathcal{U}$ to $\mathcal{W}$ and let T be the change of basis matrix from $\mathcal{U}$ to $\mathcal{Y}$. Find the change of basis matrix from $\mathcal{W}$ to $\mathcal{Y}$ and vice-versa. Briefly justify your answer.

Problem 4: Change of basis

(a) Let $w_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Show $u_1 = \begin{bmatrix} -1 \\ -3 \\ -8 \end{bmatrix}$ $u_2 = \begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$ is also a basis for $\langle w_1, w_2 \rangle$ and find the matrix A such that $\begin{bmatrix} u_1 & u_2 \end{bmatrix} = \begin{bmatrix} w_1 & w_2 \end{bmatrix} A$.

(b) Find the coordinates $\begin{bmatrix} 3 \\ 2 \end{bmatrix}_{\mathcal{U}}$ in the $\mathcal{W}$ basis. Check that your result is correct by computing in the ambient $\mathbb{R}^3$.

(c) Find the coordinates $\begin{bmatrix} 2 \\ -1 \end{bmatrix}_{\mathcal{W}}$ in the $\mathcal{U}$ basis. Check that your result is correct by computing in the ambient $\mathbb{R}^3$.

Problem 5: Define a relation on $\mathcal{M}_{m,n}$ as follows

$$A \sim B \text{ if there exist invertible matrices } S \in \mathcal{M}_m \text{ and } T \in \mathcal{M}_n \text{ such that } A = SBT$$

(a) Show that this is an equivalence relation.

(b) Show that an $n \times n$ matrix A is invertible iff A is equivalent to the identity.

(c) Can you find a set of representatives for this equivalence relation? Explain your answer. [Hint: Gaussian elimination on both sides.]