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Math 623: Matrix Analysis
Homework 3

Problem 1: Find the dimension of the subspace of $\mathcal{M}_{3,3}$ spanned by the permutation matrices.

Problem 2: Define the generalized nullspace of a matrix $A \in \mathcal{M}_{n,n}(F)$ to be the set of $x \in F^n$ such that $A^k x = 0$ for some integer k. Show that the generalized nullspace of A is a subspace of F^n.

Problem 3: Show that the simple generalization of the inclusion-exclusion principle to 3 subspaces does not hold, (sadly). Find an example in $\mathbb{R}^2$ and in $\mathbb{R}^3$ where

$$\begin{aligned}\dim(U+V+W) \neq \dim(U) + \dim(V) + \dim(W) \\ - \dim(U \cap V) - \dim(V \cap W) - \dim(W \cap U) + \dim(U \cap V \cap W)\end{aligned}$$

Problem 4: Idempotents and projections. An $n \times n$ matrix A is *idempotent* when $A^2 = I$.

(a) Prove the following theorem.
Theorem The following are equivalent (TFAE).

(1) A is idempotent.
(2) $I - A$ is idempotent.
(3) $A(I - A) = 0$.
(4) For $x \in$ colsp A, $Ax = x$.

(b) Show that if A is idempotent then $\mathbb{R}^n =$ colsp $(A) \oplus$ rtnull (A).