Abstract Algebra
Math 521A
Michael E. O'Sullivan

Review for second exam

Rings

- Know the definitions:
  - ring, commutative, identity, field;
  - unit, zero divisor, characteristic;
  - homomorphism, isomorphism.
- Know how to:
  - Prove that a subset of a ring is a subring, (or show that it isn't).
  - Prove that a function is a homomorphism, or isomorphism (or show that it isn't).
  - Show that two rings can't be isomorphic, because they have some difference in structure.
  - Identify the units and zero divisors in a ring.
- Know how to construct new rings from old and to compute in these rings.
  - The Cartesian product of rings R and S is a ring $R \times S$.
  - The 2×2 matrices over a ring R form a ring, which we write $M(R)$.
  - We also have the polynomial ring, $R[x]$ over a ring R.

Polynomial rings over a field F

- Know the special properties of $F[x]$, and that is is similar to $\mathbb{Z}$..
  - Division theorem.
  - Euclidean algorithm.
  - Prime iff irreducible.
  - Unique factorization.
- Know the relationship between roots and factors.
  - In $F[x]$, the remainder when $f(x)$ is divided by $(x - a)$ is $f(a)$.
    So $x - a$ is a factor of $f(x)$ iff a is a root of $f(x)$.
  - Know how to test whether a polynomial in $\mathbb{Z}_p[x]$ of degree 2 or 3 is irreducible.