Abstract Algebra
Math 521A
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Review for Third Exam: Chapters 5, 6

Commutative rings with identity: ideals, and congruence modulo an ideal

- Know how to
  - Define ideal, and prime ideal.
  - Prove that a subset of a ring is an ideal (or show that it isn't).
  - Use the language of ideals with $\mathbb{Z}$, $\mathbb{Z}_n$ $F[x]$, $F[x]/m(x)$.
- Know the relationship between homomorphisms and ideals.
  (The kernel is an ideal; The first isomorphism theorem; See also 6.2 #13, 24).
- Know how to compute in the quotient of a ring by an ideal.
  - If I is an ideal in R, the elements of R/I are written $a + I$ where $a \in R$.
  - $a + I = b + I$ iff $a - b \in I$.
  - Addition in R/I is defined by $(a + I) + (b + I) = (a + b) + I$.
  - Multiplication in R/I is defined by $(a + I)(b + I) = (ab) + I$.
- Know how to compute in a polynomial ring modulo a polynomial.
  - Find the inverse of an element $a(x)$ of $F[x]/m(x)$, when $a(x)$ is coprime to $p(x)$.
  - Identify units and zero divisors in $F[x]/m(x)$.
  - Identify all ideals in $F[x]/m(x)$.
  - Define irreducible and prime for polynomials.
  - Know that $m(x)$ is prime iff $m(x)$ is irreducible. In this case, $F[x]/m(x)$ is a field.
- Be able to work in $R \times S$ where R, S are commutative rings with identity.
  What are the ideals in this ring?
- Our standard examples of non-principal ideal rings.
  - Be able to compute and work with ideals in $\mathbb{Z}[x]$ and $F[x, y]$ for F a field. See 6.1#41 and example p. 140, 6.2#13, 6.3 #10.
  - Be able to find units, zero divisors, idempotent elements (x such that $x = x^2$) in $\mathbb{F}[x, y]$ modulo a simple ideal like $\langle x^2, xy, y^2 \rangle$.