NAME:

Abstract Algebra B
Math 521B
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Review for first exam

- Be able to precisely define the following terms. Be careful about the logic in the definition!
  - Group, subgroup, cyclic group, generators of a group.
  - Order of an element, order of a group.
  - Homomorphism, isomorphims, automorphism, inner automorphism.
  - Center of a group, centralizer of an element. Normalizer of a subgroup.
  - Normal subgroup, simple group.
- Here are the key theorems; be able to use them.
  - Theorem 7.8: the order of an element.
  - Theorem 7.10: properties determing a subgroup.
  - Theorems 7.18, 7.28: cyclic groups.
  - Theorems 7.26, 7.27: (Lagrange) the order of subgroups of a finite group.
  - Therorems 7.34, 7.36: Properties of normal subgroups and quotients.
  - Theorems 7.19 (and related problems), 7.41, 7.42, 7.43, 7.44: Homomorphism and isomorphism theorems.
  - Let $\phi : G \longrightarrow H$ be a homomorphism. You should be able to prove these.
    - If B is a subgroup of B then $\phi^{-1}(B)$ is a subgroup of G. In addition, if B is normal then $\phi^{-1}(B)$ is normal.
    - If A is a subgroup of G then $\phi(A)$ is a subgroup of H. If A is normal, $\phi(A)$ may not be!
  - You should be able to prove some of the simpler results about abelian groups and the order of an element (Sec. 7.2).
- Kown how to work with the standard examples.
  - $(\mathbb{Z}_n, +)$, $(U_n, *)$.
  - $\mathrm{Gl}(2, \mathbb{Q})$, $\mathrm{Sl}(2, \mathbb{Q})$ and the matrix groups over $\mathbb{Z}_p$ for p prime.
  - The symmetric group S_n, the dihedral group D_n.
  - Subgroups of the above, such as the quaternions.