

# Abstract Algebra B

## Math 521B

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Review for third exam

- Be able to define precisely the following terms. Be careful about the logic in the definition!
  - transcendental element, algebraic element, minimal polynomial (Thm. 10.6).
  - algebraic extension, simple extension, finitely generated extension, splitting field.
- Know some standard examples over  $\mathbb{Q}$  and  $\mathbb{R}$ .
  - Find the minimal polynomial of some simple examples (e.g.  $\sqrt{2+i}$ ).
  - Find bases for extensions: (e.g.  $\mathbb{Q}(\sqrt{2}, \sqrt{3})$  over  $\mathbb{Q}$ ).
  - Know the theorems and exercises about the dimension about a composite extension (Thm 10.10, Ex. 10.#7, 8, 9, 11, 13).
- Know how to work with finite fields.
  - Know the Freshman's rule (see also Ex. 10.6# 10, 12).
  - Know the existence and uniqueness theorem.
  - Know that the multiplicative group of a finite field is cyclic.
  - Know that a finite field is a simple extension of a prime field.
  - Given an irreducible polynomial whose root  $\alpha$  generates the multiplicative group of  $\mathbb{F}_{p^n}$ , construct the dictionary between powers of  $\alpha$  and polynomials in  $\alpha$ .
  - Use the dictionary to compute.