# Modern Algebra II
# Math 627B, Spring 2005

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## Some Maple commands.

Plotting curves and surfaces: You may want to use the command `with(plots);`. Then you can issue commands without the `plots[...]`.

- `plot` of `plot3d` for plotting a function.
- `plots[implicitplot]` of `plots[implicitplot3d]` for plotting the solutions of an equation.
- `plot[parametricplot]` for plotting a parametrized curve in the plane.
- `plots[spacecurve]` for plotting a parametrically defined space curve.
- `algcurves[plot_real_curve]` plotting the variety defined by a polynomial in two variables. Compare with `plots[implicitplot]`.

Handling polynomials

- `degree`, `ldegree` give the degree of the highest degree term and the lowest degree term.
- `lcoeff`, `tcoeff` give the coefficients of the highest degree term (leading term) and the lowest degree term (trailing term).
- `expand`
- `collect`
- `sort`
- `simplify`
- `convert(f, Horner)`

Computations with polynomials:

- `gcd`, `gcdex`, `lcm`
- `divide, rem, quo`
- `subs` evaluates a polynomial (or other expression at a value).
- `sum` for adding several polynomials.
- See `Help:Contents:Polynomials:entering` for a good overview.

Factorization of polynomials:

- `irreduc`
- `sqrfree` and `Sqrfree`
- `factor` and `Factor`
- `factors` and `Factors`
- `solve`, `isolve` (solutions in integers), `msolve` (solutions mod $m$), `fsolve` (for floating point solutions).
- `roots`
- `DistDeg`

Handling rational polynomials:

- `numer`, `denom`
- `normal` to simplify
- `convert(f, parfrac,x)` converting to a partial fraction, see `Help:Contents:Polynomials:Rational Expressions:partial fractions`

Creating different fields:

- `RootOf` for adjoining a root of a polynomial to a field to form an extension field. The base field can either be the rationals or $\mathbb{Z}/p$ or an extension thereof. I think this is the simplest way to handle finite fields.
- `evala` is used to evaluate expressions in algebraic extensions created using `RootOf`.
- `AFactor` to compute the factorization of a field over an algebraically closed field like $\mathbb{C}$. Used also in conjunction with `evala`
- `GF` for creating a finite field, see `Help:Contents:Packages:GF`. See also `Help:Contents:Packages:Domains` (fairly advanced).
- `diffalg[fieldextension]` for field extensions of the rationals (fairly advanced).

Functions for some important problems in computational algebraic geometry

- `algcurves[implicitize]` seems to work for hypersurfaces as well.
- `algcurves[parametrization]` for a plane curve.
- `algcurves[singularities]`
- `diff` to compute the partial derivative.

- `discrim` to compute the discriminant of a polynomial.
- `resultant` to compute the resultant of two polynomials.
- `linalg[sylvester]` and `LinearAlgebra[SylvesterMatrix]`
- `linalg[bezout]` and `LinearAlgebra[BezoutMatrix]`

Some Maple packages that may be of use

1. `Grobner`
2. `algcurves`
   - `implicitize`, `parametrization`, `singularities` are discussed above and
     `plot_real_curve` in the discussion of plots.
   - `genus` , `differentials`.
   - `Weierstraussform`, `is_hyperelliptic`, `j_invariant` for handling elliptic and hyperelliptic curves.
   - `integral_basis`, `algfun_series_sol`, `puiseaux`, `monodromy`. These have to do with maps of curves, or branches of curves (advanced!)
3. `CurveFitting` see `Spline`, `Bspine`, and `Bsplinecurve`
4. `PolynomialTools`
   - `Translate` just translates by a constant.
   - `Sort`, `Shorten`, `Shorter`. These use Mobius transformations to simplify.
   - `Splits` gives the complete factorization, and the extension field needed.
5. `Matrix PolynomialAlgebra`
6. `SNAP` has symbolic numerical algorithms for polynomials.