

Homework VI

due: Fri. 12/19/08

Clarity of exposition is crucial in this assignment. Your work should be understandable to a fellow student. You may work together to solve problems, but your solutions should be written independently.

Problem 1: Infinite algebraic extensions.

- (a) Find a sequence of fields $\mathbb{Q} \leq F_1 \leq F_2 \leq \dots$ such that each F_i is a normal extension of \mathbb{Q} . [See 6.3#5 for inspiration.]
- (b) Find a sequence of fields $\mathbb{Q} \leq F_1 \leq F_2 \leq \dots$ such that each F_{i+1} is a normal extension of F_i but not of F_{i-1} . [See 6.3#9 for inspiration.]

Problem 2: Let E_1 and E_2 be normal extensions of F .

- (a) Show that E_1E_2 (the smallest field containing E_1 and E_2) is also a normal extension of F . (See 3.5 #8.)
- (b) Let E_1 and E_2 be subfields of the Galois extension E/F . Prove that $\mathcal{G}(E_1E_2) = \mathcal{G}(E_1) \cap \mathcal{G}(E_2)$.

Problem 3: Let $E = \mathbb{Q}(i + \sqrt{2})$.

- (a) Show that i and $\sqrt{2}$ are both in E .
- (b) Find the minimum polynomial of $i + \sqrt{2}$.
- (c) Show this polynomial splits in E .
- (d) Find the Galois group and map out the relationship between subfields and subgroups for E/\mathbb{Q} .

Problem 4: Let E be the splitting field of $x^6 - 2$

- (a) Explain why the Galois group is isomorphic to D_{12} , the dihedral group on 6 elements.
- (b) Find the Galois group and map out the relationship between subfields and subgroups for E/\mathbb{Q} .

The final exam will focus on the culmination of the course: Galois' main theorem. You should be able to state the theorem and derive some straightforward consequences. The exam will be limited to finite fields, cyclotomic extensions, and roots of some simple polynomials of degree at most 4. I will give some more guidance in class during the coming week.

Please read the following problems and their solutions in Ash's text. I've grouped problems that are related. These should help you solve the assigned problems and are good preparation for the final. (I'll indicate later in the week if §6.6 will be on the final.)

- §6.1 pr. 5.
- §6.3 pr. 3,4; 5; 9-11.
- §6.4 1,2,3; 9,10.
- §6.5 1,8,9; 2; 3-7.
- §6.6 pr. 1-7.