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Math 627B: Modern Algebra I
HW 5

Due: Thursday 4/28, 2016.

Problems 1: Let ζ_n be the primitive n th root of unity $e^{2\pi i/n}$ and let $\psi_n(x)$ be its minimal polynomial.

(a) Show that $\psi_{2n}(x) = \psi_n(-x)$ for an odd integer n.

(b) Show that $\mathbb{Q}(\zeta_{2n}) = \mathbb{Q}(\zeta_n)$.

(c) Show that $\psi_{p^r}(x) = \psi_p(x^{p^{r-1}})$

Problem 2: Map out the Galois correspondence for $\mathbb{Q}(\zeta_n)$ for $n = 7, 9, 11$, as we did in class for $n = 3, 4, 5, 8$. (Why did I skip $n = 6, 10$?)

Problems 3: Let p, q be distinct primes. Find the Galois group and map out the relationship between subfields and subgroups for

(a) $\mathbb{Q}(\sqrt{p}, \sqrt{q})/\mathbb{Q}$.

Problem 4: Infinite algebraic extensions.

(a) Find a sequence of fields $\mathbb{Q} \leq F_1 \leq F_2 \leq \cdots$ such that each F_i is a normal extension of $\mathbb{Q}$. Justify your answer. [See [A] 6.3#5 for inspiration.]

(b) Find a sequence of fields $\mathbb{Q} \leq F_1 \leq F_2 \leq \cdots$ such that each F_{i+1} is a normal extension of F_i but not of F_{i-1}. [See [A] 6.3#9 for inspiration.]