DISCRETE MATHEMATICS
Math 245

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Assignments for Ch 3

Due Friday 10/15/2010

1. Using the Euclidean algorithm, find the gcd of 780 and 268.

2. It is sometimes easier to prove a statement by proving the contrapositive (which is logically equivalent to the original statement.) For the following statement, (a) Write the statement more formally the given statement, by using names for the integers and appropriate quantifiers. (b) Write the contrapositive. (c) Prove the contrapositive (it is short!).

   > If the sum of two integers is less than 50, then at least one of the integers is less than 25.

3. See Epp 2nd Ed. 3.4 #20-22 or 3rd Ed. 3.4 #27-30.
   Prove that for any integer n, $n^3 - n$ is divisible by 3. (Consider three cases.)

4. This is Epp 2nd Ed. 3.6 #14, 3rd Ed. 3.6 #26. Consider the statement,

   > For all integers a, b, c, if $a|b$ and $a \not| c$ then $a \not| (b + c)$.

   (a) Choose integers a, b, c illustrating the statement's claim.

   (b) Prove the statement using two applications of the logical equivalence $p \to (q \vee r) \equiv (p \wedge \sim q) \to r$ and a previous result that we have seen.

   (c) Prove by contradiction. Suppose a, b, c satisfy the hypothesis and the negation of the conclusion. Derive a contradiction.

5. There is a theorem that says:

   > If a prime number divides a product then it divides one of the factors.

   More explicitly:

   > For any prime number p and any integers b, c, if $p|bc$ then $p|b$ or $p|c$.

   The following statements are not true. Give a counterexample to each, and explain your counterexample in a couple of sentences.

   (a) Let a, b, c be integers with $a \neq 0$. If $a|bc$ then $a|b$ or $a|c$.

   (b) Let p be a prime number and b, c be integers. If $p|(b + c)$ then $p|b$ or $p|c$.