DISCRETE MATHEMATICS
Math 245
Michael E. O'Sullivan

Practice Exam

I. Logical arguments are based on a logic statement being a tautology. State the tautology that justifies the argument:

$$p$$
$$q$$
$$\frac{(p \wedge q) \Longrightarrow r}{r}$$

Show that it is a tautology using a truth table.
Give an example using statements in mathematical or everyday language illustrating th¡e use of this equivalence.

II. For the following statements in English:
   a) Negate the statement. (Don't just write "It is not true that ...")
   b) Translate the original statement into formal logic.
   c) Negate the formal statement and simplify. (That is, don't just write "$\sim (\ldots)$.")

   1) Everyone at the party was beautiful or smart.

   2) If a person listens to music or reads a book in the evening then s/he sleeps well.

III. Look at the knights and knaves problems and at the logic puzzles in the last section of Ch. 2

IV. Some proofs. Be organized and clear, use a Venn diagram to illustrate. There are three different approaches; try each.
   a) Use an element based argument and gradually untangle the definitions.
   b) Use an element but translate directly to a formal logic statement, use logical equivalences.
   c) Use known results about sets (distributivity, de Morgan's etc) to derive the result algebraically, without using a particular element.

   1) Let A, B, and C be subsets of a set U. Prove that $(A \cap B) - C = (A - C) \cap (B - C)$.

   2) If A, B, and C are sets such that $A \subseteq B$, and $A \subseteq C$, then $A \subseteq B \cap C$.