DISCRETE MATHEMATICS
Math 245

Michael E. O'Sullivan

Assignments for Ch 3

Due Mon 3/8/10

1. Prove the following statement.

   Let a, b, c, r, s be integers with $a \neq 0$. If $a|b$ and $a|c$ then $a|br + cs$.

2. There is a theorem that says:

   If a prime number divides a product then it divides one of the factors.

   More explicitly:

   Let p be a prime number and b, c be integers. If $p|bc$ then $p|b$ or $p|c$.

   The following statements are not true. Give a counterexample to each, and explain your counterexample in a couple of sentences.

   (a) Let a, b, c be integers with $a \neq 0$. If $a|bc$ then $a|b$ or $a|c$.
   (b) Let p be a prime number and b, c be integers. If $p|(b + c)$ then $p|b$ or $p|c$.

3. See Epp 2nd Ed. 3.4 #20-22 or 3rd Ed. 3.4 #27-30.
   Prove that for any integer n, $n^3 - n$ is divisible by 3. (Consider three cases.)

4. Prove the following result using the definition of floor (See Epp 2nd Ed. and 3rd Ed 3.5 #23-24. Do not use #23 to prove this problem.) Let x be a real number. If x is not an integer then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$.

Due Mon. 3/15/10

1. It is sometimes easier to prove a statement by proving the contrapositive (which is logically equivalent to the original statement.) For the following statement, (a) Write the contrapositive. Be more explicit than the given statement, by using names for the integers. (b) Prove the contrapositive (it is short!).

   If the sum of two integers is less than 50, then at least one of the integers is less than 25.

2. This is Epp 2nd Ed. 3.6 #14, 3rd Ed. 3.6 #26. Consider the statement,

   For all integers a, b, c, if $a|b$ and $a \not| c$ then $a \not| (b + c)$.

   (a) Choose integers a, b, c illustrating the statement's claim.
   (b) Prove the statement using the logical equivalence $p \to (q \vee r) \equiv (p \wedge \sim q) \to r$ and a previous result that we have seen.
   (c) Prove by contradiction. Suppose a, b, c satisfy the hypothesis and the negation of the conclusion. Derive a contradiction.

3. Mimic the proof that $\sqrt{2}$ is irrational to prove that $\sqrt{3}$ is irrational.

4. We know that the sum of two rational numbers is rational. Prove by contradiction that the sum of a rational and an irrational is irrational. (The proof is easily altered to show that the product of a nonzero rational and an irrational is irrational.)