DISCRETE MATHEMATICS
Math 245
Michael E. O'Sullivan

Suggestions for Preparing for the Final Exam

I. Understand fundamental logic:

- Truth tables for $\wedge$, $\vee$, $\Longrightarrow$, xor.

- Be able to show logical equivalence using a truth table.

- Know some important equivalences
  (Thm. 1.1.1 and $p \Longrightarrow q \equiv \sim p \vee q$ and $p \Longrightarrow (q \vee r) \equiv (p \wedge \sim q) \Longrightarrow r$).

- Know how to negate a statement (Important!).

- Pay particular attention to quantifiers, existential ($\exists$) and universal ($\forall$).

II. Know the basics of set theory!

- Definitions of subset, intersection, union, set difference, complement.

- Definitions of power set, Cartesian product, partition.

- Be able to prove one set is a subset of another.

III. Know the basics of number theory.

- The definition of *divides* and its properties.

- Be able to state and use the Quotient-Remainder Theorem.

- Be able to state and use the Unique Factorization Theorem.

- Know the classic proofs by contradiction

  - There exist an infinite number of primes.
  - $\sqrt{p}$ is irrational for p prime.
  - The sum of a rational number and an irrational number is irrational.

IV. Functions and relations.

- Know the definitions: Relation, inverse of a relation, function. For functions: injective (one-to-one), surjective (onto) and bijective.

- Be able to determine when a relation is a function, and if so, when it is injective, surjective, or bijective.

- Be able to work with lists, tables, arrow diagrams and formulas to define relations.

- Be able to find the inverse of a bijective function (e.g. $f(x) = 2x + 7$).

- Give examples of functions satisfying various properties
  (7.3 #4-10 (2nd Ed.) 7.2 #5-14 (3rd Ed.)).

V. Relations on a set.

- Know how to use a table, a list of elements or an arrow diagram to represent a relation on X. (The arrow diagram is different from that for a relation from X to Y).

- Verify or prove that a relation R is reflexive (ditto for symmetric, transitive, antisymmetric, an equivalence relation, or a partial order).

- For a relation R on A, be able to find the smallest relation containing R which is symmetric (ditto for reflexive, transitive, an equivalence relation, a partial order).

- Know the standard examples of equivalence relations
  (mod n, 10.3.10 and exercises 10.3 #18, 19, 22, 23 3rd Ed., 10.3 #15, 16, 19, 20 2nd Ed.).

- Know the standard examples of partially ordered sets: $\leq$ for the integers (or rationals) divides on the integers; $\mathcal{P}(\mathcal{A})$ for a set A; D_n; (10.5 #16, 17, 18, 19, 20, 21 both Eds.).

- Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

VI. Know the basics of recursion and induction!

- State the well-ordering principle.

- State the principle of induction.

- Be able to use summation and product notation.

- Find the first several terms of a sequence given the initial terms and the recurrence formula.

- Know the formulas for the following sums:

  - The sum of a geometric sequence.
  - The sum of the first n integers.

VII. Know how to prove by induction!

- Use full sentences.

- State the predicate.

- Prove the base step.

- State the assumption for the inductive step.

- Do the inductive step.

VIII. Know how to count!

- Know the 4 ways to choose and the formulas for 3 of them (I won't test "order unimportant, repetition allowed").

- State the binomial theorem, and use it to find a particular coefficient in a binomial expansion (6.7 #4, 8).

- Poker hands (I will describe the hand, and I may give you a strange deck).