## DISCRETE MATHEMATICS Math 245 Michael E. O'Sullivan

## Suggestions for preparing for the Fourth Exam

(\*) indicates level 2 problems.

I. Sequences and recursion.

- Be able to use summation and product notation.
- Be able to use recursive formulas.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.
- Check that a sequence that is given explicitly satisfies a recurrence formula.

II. Know the formulas for the following sums:

- The sum of a geometric sequence. For any real  $r \neq 1$ , and for any  $n \in \mathbb{N}_0$ ,  $\sum_{i=0}^n r^i = \frac{r^{n+1}-1}{r-1}$ .
- The sum of the first n integers. For any integer  $n \ge 1$ ,  $\sum_{i=1}^{n} = \frac{n(n+1)}{2}$ .

## III. Know how to prove by induction!

- To write an induction proof:
  - Use full sentences.
  - State the predicate.
  - Prove the base step.
  - State the assumption for the inductive step.
  - Do the inductive step.
- For level 1:
  - I will guide you through the proof.
  - There are three types of proofs.
    Proving the nth element of a sequence is equal to some formula involving n (§5.2 #10-16).
    Proving an inequality (§5.3 #8-15).
    Proving a divisibility result (§5.3 #16-20).
- \* For level 2:
  - You should be able to write the proof yourself.
  - Be able to use strong induction (§5.4 #1-6).