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DISCRETE MATHEMATICS
Math 245
Michael E. O'Sullivan

Assignments for Ch 4

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Due Thursday 03/08/2012

1. It is sometimes easier to prove a statement by proving the contrapositive (which is logically equivalent to the original statement.) For the following statement, (a) Write the statement formally, by using names for the integers and appropriate quantifiers. (b) Write the contrapositive. (c) Prove the contrapositive (it is short!).

   > If the product of two positive integers is less than 100, then at least one of the integers is less than 10.

2. Using the Euclidean algorithm, find the gcd of 780 and 268.

3. Exercises with representations of integers.
   (a) Convert 451 in base 10 to binary and to octal.
   (b) Convert 101110_2 and 535_8 to base 10.
   (c) Make a multiplication table for one digit numbers base 7.
   (d) Find the product in octal, $36_7 * 74_7$.

4. See Epp 2nd Ed. 3.4 #20-22 or 3rd and 4th Ed. 3.4 #27-30.
   Prove that the product of two consecutive integers is not 1 mod 3. (So the product is either divisible by 3 or equal to 2 mod 3.) Consider three cases determined by the quotient-remainder theorem.)

5. There is a theorem that says:

   > If a prime number divides a product then it divides one of the factors.

   More explicitly:

   > For any prime number p and any integers b, c, if $p|bc$ then $p|b$ or $p|c$.

   The following statements are NOT true. Give a counterexample to each, and explain your counterexample in a couple of sentences.

   (a) Let a, b, c be integers with $a \neq 0$. If $a|bc$ then $a|b$ or $a|c$.
   (b) Let p be a prime number and let b, c be integers. If $p|(b + c)$ then $p|b$ or $p|c$.