Math 241: Workshop on Geometry Software

Days 3 and 4, 2015-09-08, 2015-09-10

Conics

Here are the classical definitions of conics.

- **Parabola**: Let F be a point and ℓ a line not containing F. The set of points that are equidistant from F and ℓ is called a *parabola*. The point F is called the *focus* and the line ℓ is called the *directrix* of the parabola.

- **Ellipse**: [From Wikipedia] In mathematics, an ellipse is a curve on a plane surrounding two focal points such that the sum of the distances to the two focal points is constant for every point on the curve.

  [More precisely] Let F and G be two points and let $d > FG$ be a real number. The set of points P such that $PF + PG = d$ is called an *ellipse*. The points F and G are the foci of the ellipse, and FG/d is the *eccentricity*.

- **Hyperbola**: [From Wikipedia] A hyperbola is the locus of points where the absolute value of the difference of the distances to the two foci is a constant equal to $2a$, the distance between its two vertices.

  [More precisely] Let F and G be two points and let $d < FQ$ be a positive real number. The set of points P such that $|PF - PG| = d$ is called an *hyperbola*. The points F and G are the foci of the hyperbola, and FG/d is the eccentricity.

Use ruler and compass constructions (and the composite constructions in Geogebra that we discussed last time) to construct each of these conic sections. Use the trace option and animation to make it lively.

Transformations

Geogebra has several built-in tools for transformations of the plane.

- *Dilation*: Fixes one point (the origin for the dilation) and expands the distance to that point by some consant factor.

- *Translation*: Shifts all coordinates by some fixed vector (a, b).

- *Rotation*: Fixes one point and rotates all other points by a fixed angle.

- *Reflection*: This may be done through a line (the everyday notion of reflection), or through a point, or, most interesting, through a circle.