Math 241: Workshop on Geometry Software

Day 9, 2015-10-13

Exponential and Logarithmic Functions

Here are some exercises to help develop your intution for exponential functions.

- Graph $f(x) = 2^x$.
- Compare the graph of $f(x)$ with graphs for the functions $g(x) = f(x - p)$, $h(x) = f(x) + q$, $k(x) = f(a * x)$, $m(x) = c * f(x)$, exploring how different values of p, q, c, a affect the graphs of the corresponding functions.
- Show graphically that the function $m(x)$ can also be represented in the form used in $g(x)$. Then prove it using rules of exponents. Graph $f(x) = 2^x$ and $m(x) = c * f(x)$, using a slider for c. Define p to be computed from c by the formula above. Show that for this value of p, $g(x) = f(x - p)$ has the same graph as $m(x)$.
- A function like $k(x)$ can also be expressed as an exponential of x with a different base using $a = log_2 t$ and rules of exponents. Show this graphically. Graph $f(x) = 2^x$ and $k(x) = f(a * x)$, using a slider for a. Define b to be computed from a by the formula above. Show that for this value of b, the graph of b^x is the same as the graph of $k(x)$. (Use highlighting!)

Carbon Dating

Take a look at the wikipedia page for carbon dating. Roughly, here is how it works. The assumption in carbon dating (in its rough form) is that when a living being dies, the proportion of Carbon-14 is the same as that in the atmosphere (this is not true but is close enough for rough estimates). After that point, the the Carbon-14 decays with half-life about 5730 years, and the other forms of Carbon are stable. Consequently, the proportion of Carbon-14 in a dead being declines over time. This proportion can be used to infer the time since death. The proportion of carbon dioxide is extremely small, $1.5/10^{12}$. That proportion will reduce by a factor of 2 in roughly 5730 years.

Assignment

- Make a worksheet in which you present the following toy carbon-dating problem. On planet Frisbee, the proportion of Carbon-14 is 10% of the total carbon in the atmosphere and the rest is Carbon-12. An initial sample has 10,000 atoms of carbon. Assume carbon has a half-life of 6000 years (it is fairly close to that amount). Give the formula for the total amount of carbon-14 in the sample as a function of time. Illustrate how one would use a measurement of the amount of the carbon-14 in the sample (many years later) to figure out how much time has elapsed since the sample was created. Pretend you have discussed exponential functions with your class and explain how the graph relates to finding the age of the object.